IB Math Applications and Interpretations Summer Assignment

Course Title: IB Math: Applications and Interpretations (formerly IB Math Studies)

Teacher Name: Lianne Jaramillo

Teacher contact information: lianne.jaramillo@apsva.us

Purpose of Assignment: The following summer assignment is designed to prepare you for the forthcoming IB Math course. The IB MAI course is rigorous and fast paced and has a lot of content to cover before the exams in May. This summer assignment contains content that has already been covered in previous Algebra classes but must be mastered before entering the course. The assignment is designed to allow you to review the material previously covered in math courses so that you will be well prepared with several ideas that occur throughout the IB Math course.

Estimated time to complete the assignment: On an average it will take 4-6 hours to complete the assignment.

Instructions for Assignment: There are two parts to this assignment

PART I: IA (Internal Assessment Project) Readiness

The Internal Assessment (IA) is an IB required assessment and will count toward the overall score earned from IB. It is your chance to apply your understanding of IB Math A&I concepts and skills to real world situations. More attention and focus on this project will improve the overall IB score.

Possible Topic

You should start **thinking** of a creative and serious idea for your **IA Project**.

About the IA: The IA is a piece of written work which will be based on your personal research from a wide variety of topics including, but not limited to investigations, applications, statistical studies and surveys. **This IA will be done during the course of the year.** The project is an independent research and data gathering project for which you will collect, prepare, organize, analyze, and evaluate data.

Fill out the "Initial Planning"

During the summer, you must come up with several different topics that you wish to pursue or explore. Refer to the attached information to peruse through some ideas of IA topics. Write down your ideas in PART 1 below.

Students have looked at comparing two country values (GDP vs birth mortality rate), predicting a trendline (such as the spread of a virus), etc. There are lots of ideas, but if you pick a topic that you care about then the project tends to be much easier.

Part II: Algebra 3 Review Worksheets

All work must be done neatly on your **own paper.** Make sure each page of your work is labeled with the corresponding Worksheet Number and Topic Name. Answers to problems must be circled to facilitate grading. Most importantly, the work should be neat!

You should have your own graphing calculator (TI-84 recommended) for this course.

Due date and Method of assessment:

Part 1: Possible Project Ideas – Due the 2nd week of school.

Part 2: A quiz will be given over the material covered on the review worksheets. The summer assignment quiz will occur the 2nd week of school

Thank you.
Have a great summer! Looking forward to seeing you soon. ☺

Ms. Jaramillo

PART 1 – Possible Project Ideas

The project has a component called "Personal Engagement". It is important that you care about the subject that you are exploring so that this naturally shines in your project.

You must show some mathematics and that you understand the mathematics. We are going to focus on our projects on modeling (predictions) and statistics.

Complete at least 3 ideas below:

Possible Topic	**Why you want to explore it**	**What is a possible data source?**

<u>Worksheet 1 – Exponent Properties</u>

Review:

1. $a^n * a^m = a^{n+m}$
2. $\left(a^n\right)^m = a^{nm}$
3. $\frac{a^n}{a^m} = a^{n-m}$
4. $a^0 = 1\ (a \neq 0)$
5. $a^{-n} = \frac{1}{a^n}$
6. $(ab)^n = a^n b^n$

Practice:

1. $x^2 \bullet x^3$
2. $\left(2k^3\right)\left(-4k^4\right)\left(3k^{-2}\right)$
3. $\left(-2x^3\right)^2$
4. $-\left(2x^3\right)^2$
5. $\left(-2x^2\right)^3$
6. $-\left(2x^2\right)^3$
7. x^{-3}
8. $4x^{-3}$
9. $\frac{3}{x^{-2}}$
10. $\frac{-5}{x^{-4}}$
11. $\frac{x^8}{x^2}$
12. $\frac{x^3}{x^6}$
13. $\frac{x^{-3}}{4x^5}$
14. $\frac{-10x^{15}}{5x^{-3}}$
15. $x^2 \bullet x^{-2}$
16. x^0
17. $\left(\frac{4x^2}{5y}\right)^3$
18. $\left(3y^2\right)\left(2y^{21}\right)$
19. $\left(4x^3y^2\right)\left(-3xy\right)$
20. $\left(-2st^5\right)\left(-4st^{-3}\right)$
21. $\left(5a^2b^3\right)\left(a^{-2}b\right)$
22. $\left(-\frac{a^{-3}}{3a^{-1}b}\right)^4$
23. $\frac{3}{4d} \cdot \frac{(2d)^4}{c^3}$
24. $y^0\left(8x^6y^{-3}\right)^{-2}$
25. $\left(5r^5\right)^3 \cdot r^{-2}$

<u>Worksheet #2 – Trigonometry Review</u>

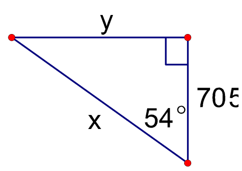
Review:

Make sure your calculator is in degree mode. We saw in Geometry that $sin\,(ANGLE) = RATIO$. The trigonometry functions relate sides of of a right triangle. You probably saw the mnemonic:

<u>SOH CAH TOA</u>

$sin \;=\; \frac{opposite}{hypotenuse} \qquad cos \;=\; \frac{adjacent}{hypotenuse} \qquad tan \;=\; \frac{opposite}{adjacent}$

Ex 1. Solve for the variables



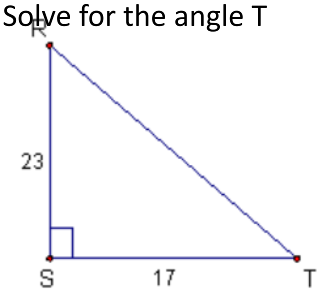
$tan\,54° \;=\frac{y}{705} \qquad cos\,54° \;=\frac{705}{x}$

$705 \,*\, tan\,54° \,=\, y \quad x \,*\, cos\,54° \,=\, 705$

$x \,=\frac{705}{cos\,54°}$

Now type into the calculator

Ex 2. Solve for the angle T



$tan\,T \;=\frac{23}{17}$

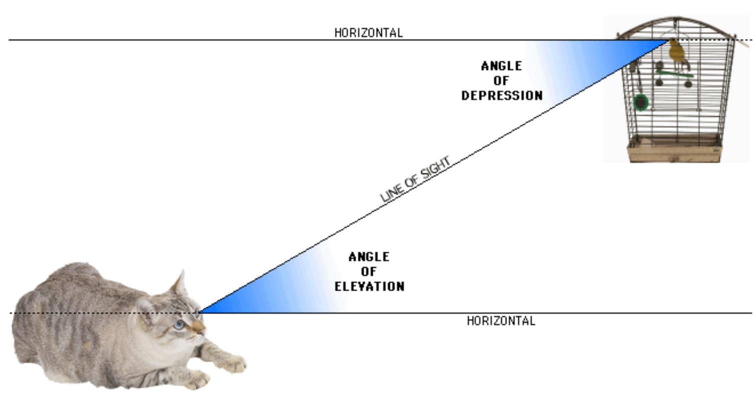
$T \;=\left(\frac{23}{17}\right)$

Now type into calculator

<u>Angles of Elevation and Depression</u>:

Angle of Elevation – The angle that looks upwards from a horizontal between two objects

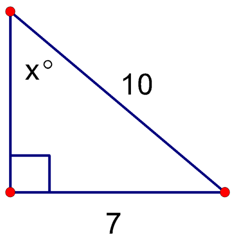
Angle of Depression – The angle that looks downwards from a horizontal between two objects.



Practice:

<u>Questions 1-9</u>: Find the value of the variable to the nearest tenth.

1.



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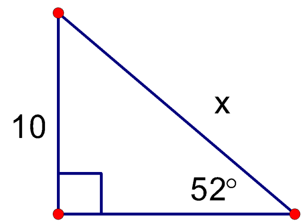

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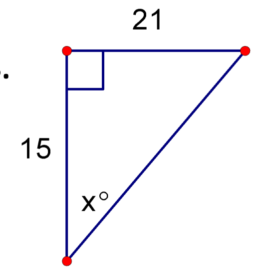

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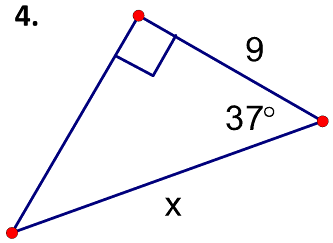

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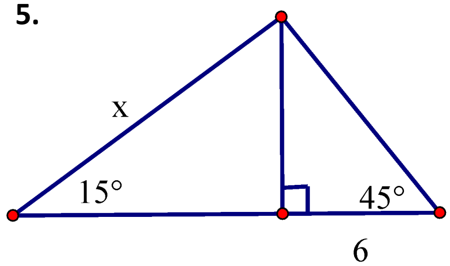

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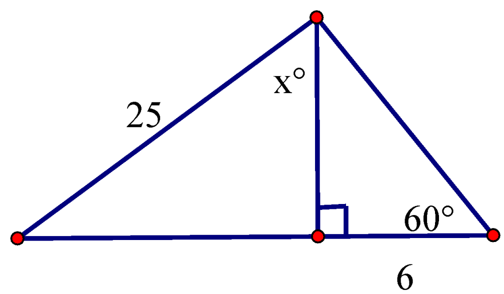

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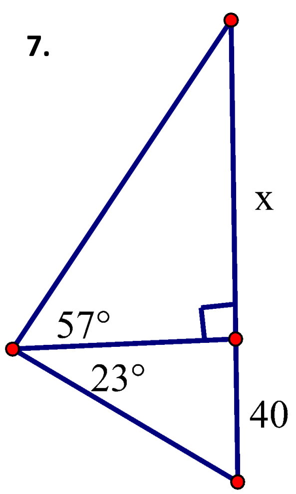

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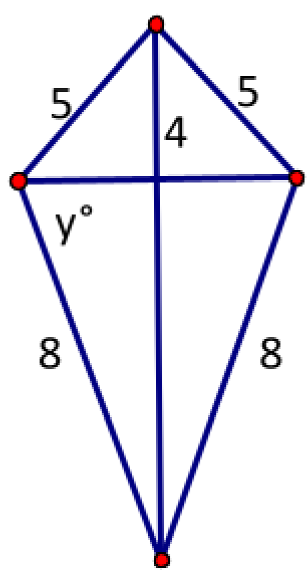

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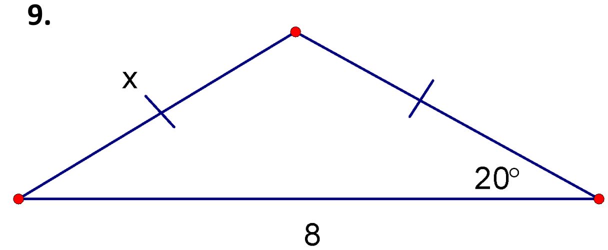


<u>Questions 10-13</u>: Accurately draw a picture representing the problem and answer the question.

10. A building casts a shadow 40 feet long when the sun's angle of elevation is 58°. Find the height of the building to the nearest foot.

11. A forest ranger watches for fires from a look-out tower built on a high hill. The site of the tower is 740 m above most of the surrounding land, and the tower itself is 24 m tall. If the ranger sights a fire at an angle of 7° (hint: it is an angle of depression), how far, to the nearest meter, is the fire from the top of the tower?

12. When the sun's angle of elevation is 42°, a tree casts a shadow 17 m long. How tall is the tree to the nearest meter?

13. The angle of depression from the top of a tower to a point A is 23°. The distance from *A* to the base, *B*, of the tower is 80 m. How tall is the tower to the nearest meter?

Worksheet #3 – Systems of Equations

Review:

Substitution

Example: $\begin{cases} y = 3x + 2 \\ x + 2y = 11 \end{cases}$

Since we have y =, replace y in the 2nd equation to get: $x + 2(3x + 2) = 11$

Now we distribute and combine like terms: $x + 6x + 4 = 11 \rightarrow 7x + 4 = 11$

Solve for x: $7x = 7; \quad x = 1$

Now substitute back into y = 3x+2 to find y: $y = 3(1) + 1 = 4$

Write your answer: (1, 4)

Linear Combinations (Elimination):

Example: $\begin{cases} 6x + 5y = 19 \\ 2x + 3y = 5 \end{cases}$

The goal is to add (or subtract) the two equations so one variable is removed.

Multiply the 2nd equation by -3 to get: $6x + 5y = 19 \ - 6x - 9y =- 15$

Add the two equations together: $- 4y =- 4$

Solve for y: $y = 1$

Substitute into either original equation to get x: $2x + 3(1) = 5$
$2x = 2$
$x = 1$

Note sometimes you need to multiply both equations by a number

Graphing:

Isolate the variable y in each equation and type those equation in the y= feature on your graphing calculator. Then hit the *2ND* button then the *TRACE* button then option #5: *intersect*.

Practice: Solve by any method, but make sure you understand all three methods.

1. $\begin{cases} y = 3x + 4 \\ y = -2x - 1 \end{cases}$

2. $\begin{cases} 9x + 2y = 39 \\ 6x + 13y = -9 \end{cases}$

3. $\begin{cases} 8x - 7y = -3 \\ 6x - 5y = -1 \end{cases}$

4. $\begin{cases} x + y = 3 \\ x - y = 5 \end{cases}$

5. $\begin{cases} y = 2x - 4 \\ -6x + 3y = -12 \end{cases}$

6. $\begin{cases} 3x - 2y = 3 \\ -x + y = 1 \end{cases}$

7. $\begin{cases} 2x + y = -15 \\ y - 5x = 6 \end{cases}$

8. $\begin{cases} x + y = -1 \\ -2x + y = -7 \end{cases}$

9. $x = 2(y+1)^2 - 6$
   $x + y = 3$

10. $y = x^2 - 3$
    $x^2 + y^2 = 9$

11. $y^2 - x^2 + 3y = 26$
    $x^2 + 2y^2 = 34$

12. $y^2 = 10 - 6x^2$
    $4y^2 = 40 - 2x^2$

<u>Worksheet #4 – Domain and Inverses</u>

Review:

Domain – the possible set of input values (often associated with the x-axis)

Range – the set of output values (often associated with the y-axis)

Relation – a set of ordered pairs

Function – a relation where each input value corresponds to exactly one output value. (this means for every x-value there is only one possible y-value).

Inverses – reverses or undoes the effect of a relation/function.

IB expects you to know that if the point (a, b) is on a relation, then the point (b, a) is on its inverse. This is what you saw in Algebra two when you switched the x and y values, then solved for y to create the inverse. Since we are switching the input and output values, this means that the domain of the function will be the range of its inverse.

Notation: The inverse of a function *f(x)* is written as $f^{-1}(x)$, and $f(a) = b,\ then\ f^{-1}(b) = a$.

This also means that the inverse is a reflection in the line y = x of the original.

Example:

1. If $f(-\ 10) = 5$, then find $f^{-1}(5)$.

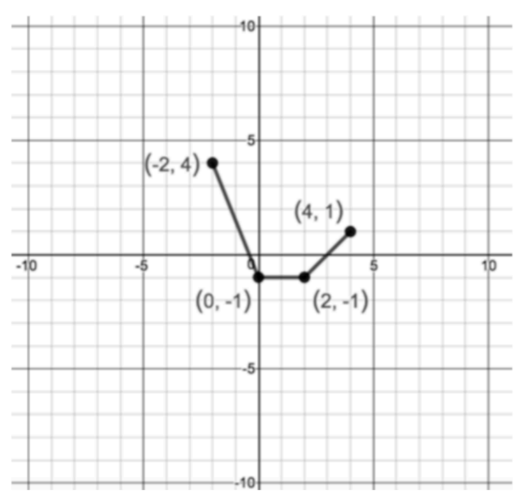
$f(-\ 10) = 5$ represents the point (-10, 5)

The inverse of this point is (5, -10)

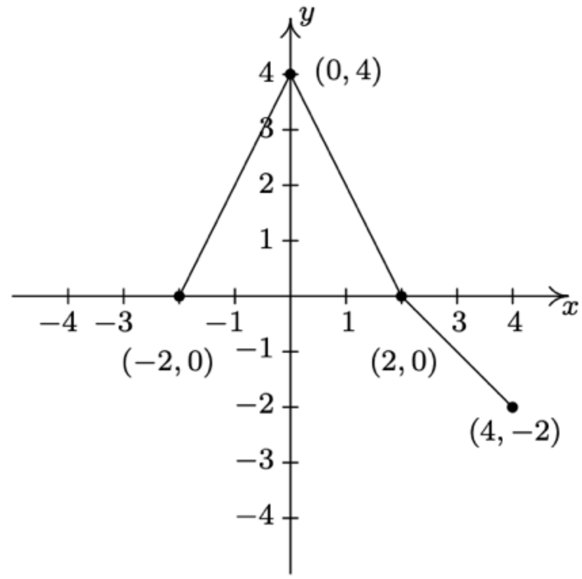
This means $f^{-1}(5) =-\ 10$.

Practice:

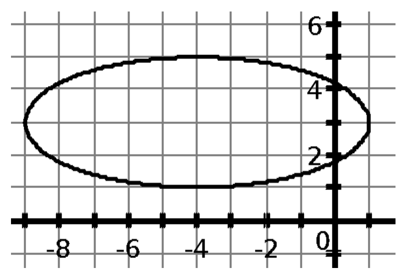
1. a. Is the graph a function?
b. What is the domain?
c. What is the range?
d. On the same graph, sketch the inverse.
e. What is the domain of the inverse?



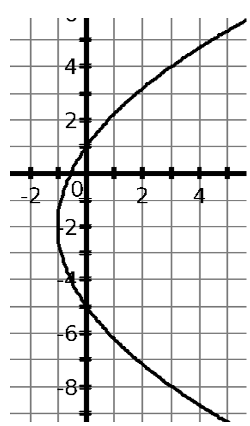
2. a. Is the graph a function?
b. What is the domain?
c. What is the range?
d. On the same graph, sketch the inverse.
e. What is the domain of the inverse?



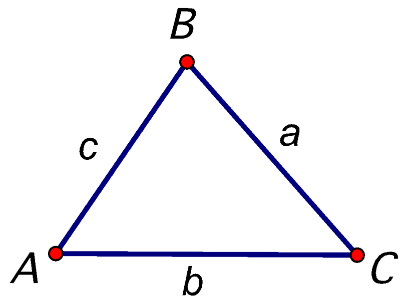
3. a. Is the graph a function?
b. What is the domain?
c. What is the range?
d. On a new graph, sketch the inverse.
e. What is the domain of the inverse?
f. Is the inverse a function?



4. a. Is the graph a function?
b. What is the domain?
c. What is the range?
d. On a new graph, sketch the inverse.
e. What is the domain of the inverse?
f. Is the inverse a function?



Worksheet #5 – Sine Rule and Area of a Triangle



The sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

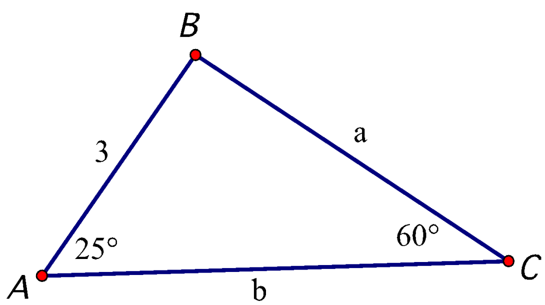
Area of a triangle: $A = \frac{1}{2}ab\sin C$

(need two sides and the

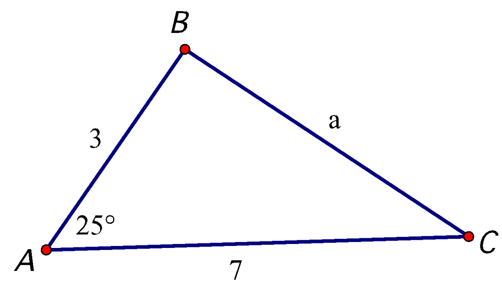
You will not have the ambiguous case for this class, there will be only one possible solution for each angle when solving with the sine rule

Example: and the area of the triangle.

1. Solve the triangle (find all missing sides and angles)



2. Find the area of the triangle



1. Find angle B first

$B = 180° - 25° - 60° = 95°$

2. Set up sine rule to find side b,

$\frac{b}{\sin 95} = \frac{3}{\sin 60}$

3. Cross multiply

$b * \sin 60 = 3\sin 95$

4. Divide by sin 60, and enter into calculator

$b = \frac{3\sin 95}{\sin 60}$

5. To find a use the equation $\frac{3}{\sin 60} = \frac{a}{\sin 25}$

6. Solve in the same way to get $a = \frac{3\sin 25}{\sin 60}$.

1. $A = \frac{1}{2}ab\sin C$

(this is just the general form it can be rewritten)

$A = \frac{1}{2}bc\sin A$

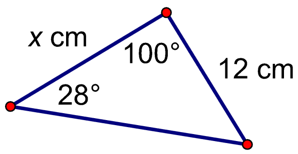
2. $A = \frac{1}{2}(3)(7)\sin 25$

Enter this into the calculator.

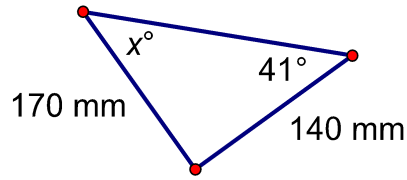
Practice:

1. Solve for the variable

a.

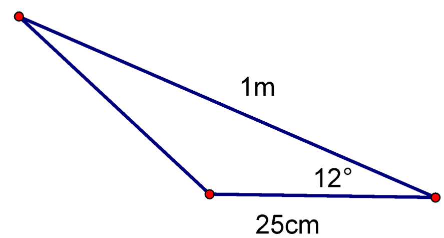


b.

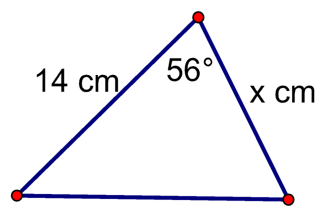


2. Solve the triangle given a = 23, b = 11, and A=122°

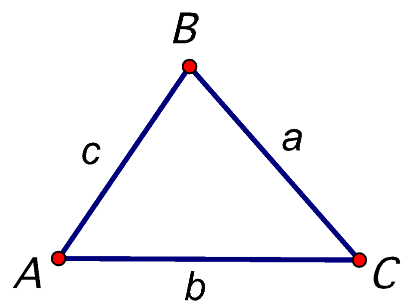
3: Find the area of the triangle to the nearest cm.



4: The triangle has 130 cm^2 as its area. Find the value of x.



Worksheet #6: Cosine Rule



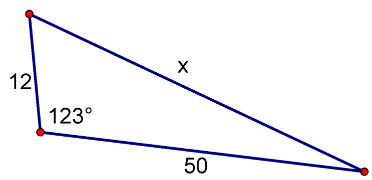
This has two forms:

Find a missing side: $a^2 = b^2 + c^2 - 2bccos\,A$

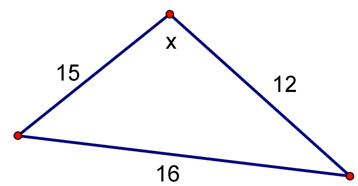
Find a missing angle: $cos\,A = \frac{b^2+c^2-a^2}{2bc}$

Example: Find the value of x

a.



b.



1. Fill-in $a^2 = b^2 + c^2 - 2bccos\,A$

$x^2 = 12^2 + 50^2 - 2(12)(50)cos\,123$

2. Now put into the calculator to get:

$x^2 = 2970.7834....$

3. Take the square root

1. Fill-in cos $cos\,A = \frac{b^2+c^2-a^2}{2bc}$

$cos\,x = \frac{15^2+12^2-16^2}{2(15)(12)}$

2. Simplify the fraction side to get:

$cos\,x = \frac{113}{360}$

3. Use the cos^{-1}

$x = (\frac{113}{360})$

And enter into the calculator.

Practice: Sketch the triangle and solve for the indicated angle or side.

1. $a = 4,\ b = 5,\ c = 6.$ Find angle C.

2. $a = 8,\ b = 12,\ c = 11.$ Find angle A.

3. $A = 40°,\ b = 5,\ c = 6.$ Find side a.

4. $B = 140°, a = 13,\ c = 16.$ Find side b.

5. $a = 18,\ b = 16,\ c = 10.$ Find angle B.

Worksheet #7: Quadratics

Review:

The standard form of a quadratic function is:

$$y = ax^2 + bx + c$$

- ★ The parabola opens *up* if a > 0 and opens *down* if a < 0
- ★ The axis of symmetry is the vertical line $x = \frac{-b}{2a}$
- ★ The vertex lies on the axis of symmetry. $(\frac{-b}{2a}, f(\frac{-b}{2a}))$

The Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Factoring:

Differences of Squares: $a^2 - b^2 = (a + b)(a - b)$

Box Method

$\underset{a}{2x^2} + \underset{b}{7x} + \underset{c}{5}$

Step 1: Find the product (ac) =10

Step 2: Find the sum (b) =7

Step 3: Rewrite

5 and 2

$2x^2$	$5x$
$2x$	5

Step 4: Factor out a GCF (work backwards to match)

	$2x$	$+5$
x	$2x^2$	$5x$
$+1$	$2x$	5

Step 5: Write in factored form.

$(2x+5)(x+1)$

Factor by Grouping

$2x^2+7x+5$

$(2x^2+5x) + (2x+5)$

$x(\underline{2x+5}) + 1(\underline{2x+5})$

$(2x+5)(x+1)$

Practice:

Find the vertex, y-intercept, x-intercept, line of symmetry, and tell if it opens up or down.

1. $y = -2x^2 + 12x - 7$

2. $y = \frac{1}{4}x^2$

3. $y = 4x^2 + 7$

4. $y = -x^2 - 2x - 3$

Find the zero(s) of the equation.

5. $f(x) = x^2 - 2x - 3$

6. $h(x) = x^2 + x + 1$

7. $g(x) = x^2 + 6x - 7$

8. $k(x) = -x^2 + 6x - 9$

Solve the equation. Note some may require the Quadratic Formula.

9. $2x^2 = 8$

10. $-10 = r^2 - 10r + 12$

11. $3x^2 - 11 = 7$

12. $4z^2 = 9$

13. $7c^2 = 100$

14. $3x^2 + 5x = 8$

15. $2x^2 - 7 = x$

16. $3g^2 - 6g - 14 = 3g$

17. $6z^2 = 2z^2 + 7z + 5$

18. $-4y^2 - 3y + 3 = 2y + 4$

19. $(x + 13)^2 = 25$

20. $-2x^2 = -32$