AP Calculus AB/IB Math A&A SL2:   Summer Assignment

Course Title: AP Calculus AB/IB Math A&A SL2
Teacher name: Sukhdeep Kaur, Karen Hatchl, and Julianna Carousso
Teacher contact information: sukhdeep.kaur@apsva.us
karen.bui@apsva.us
julianna.carousso@apsva.us

Purpose of Assignment: To master the prerequisite content that is necessary for you to understand prior to taking AP Calculus AB.

Estimated time to complete Assignment: No more than 6 hours

Due date and method of assessment for Assignment: Due in the first class of the second week of school. Although this is due in the first class of the second week of school, we will be taking questions on the summer assignment on the first full day of class (if needed). There will be a summer assignment assessment (non-calculator) in the second week of school.

Instructions for Assignment:

- Read Section 1.2
- Do the circled problems from Section 1.2
- Complete the Graphing Sprint worksheet
- Read Section 1.6
- Do the circled problems from Section 1.6
- Complete the Solving Review worksheet
- Fill out the unit circle worksheet (front and back). Do as much as you can from memory or from deriving by reference triangle. It is expected that you have the unit circle memorized by the time you reach Calculus.
- Complete the Unit Circle Practice worksheet

## 1.2 NOTES

Investigating Equations of Lines
Use a graphing utility to graph each of the linear equations. Which point is common to all seven lines? Which value in the equation determines the slope of each line?

a. $y - 4 = -2(x + 1)$

b. $y - 4 = -1(x + 1)$

c. $y - 4 = -\frac{1}{2}(x + 1)$

d. $y - 4 = 0(x + 1)$

e. $y - 4 = \frac{1}{2}(x + 1)$

f. $y - 4 = 1(x + 1)$

g. $y - 4 = 2(x + 1)$

Use your results to write an equation of a line passing through $(-1, 4)$ with a slope of m.

Equations of Lines

Any two points on a nonvertical line can be used to calculate its slope. This can be verified from the similar triangles shown in Figure 1.14. (Recall that the ratios of corresponding sides of similar triangles are equal.)

$$m = \frac{y_2^* - y_1^*}{x_2^* - x_1^*} = \frac{y_2 - y_1}{x_2 - x_1}$$
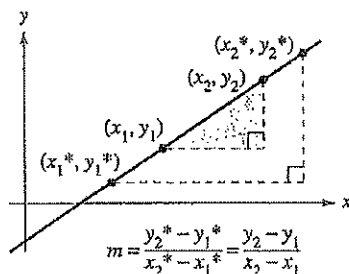
Any two points on a nonvertical line can be
used to determine its slope.
Figure 1.14

You can write an equation of a nonvertical line if you know the slope of the line and the coordinates of one point on the line. Suppose the slope is m and the point is (x_1, y_1). If (x, y) is any other point on the line, then
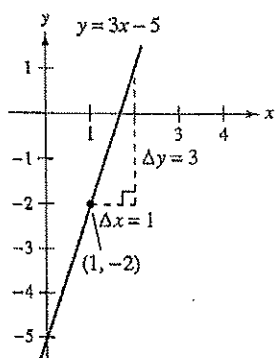
$$\frac{y - y_1}{x - x_1} = m.$$

This equation, involving the two variables x and y, can be rewritten in the form $y - y_1 = m(x - x_1)$, which is called the **point-slope form of the equation of a line.**

POINT-SLOPE FORM OF THE EQUATION OF A LINE

An equation of the line with slope m passing through the point (x_1, y_1) is given by

$$y - y_1 = m(x - x_1).$$

EXAMPLE 1 Finding an Equation of a Line

Find an equation of the line that has a slope of 3 and passes through the point $(1, -2)$.

Solution

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
$$y - (-2) = 3(x - 1) \qquad \text{Substitute } -2 \text{ for } y_1, 1 \text{ for } x_1, \text{ and } 3 \text{ for } m.$$
$$y + 2 = 3x - 3 \qquad \text{Simplify.}$$
$$y = 3x - 5 \qquad \text{Solve for } y.$$

(See Figure 1.15.)

The line with a slope of 3 passing through
the point $(1, -2)$
Figure 1.15

NOTE Remember that only nonvertical lines have a slope. Consequently, vertical lines cannot be written in point-slope form. For instance, the equation of the vertical line passing through the point $(1, -2)$ is $x = 1$.

1.2 NOTES

Ratios and Rates of Change

The slope of a line can be interpreted as either a *ratio* or a *rate*. If the *x*- and *y*-axes have the same unit of measure, the slope has no units and is a **ratio**. If the *x*- and *y*-axes have different units of measure, the slope is a rate or **rate of change.** In your study of calculus, you will encounter applications involving both interpretations of slope.

EXAMPLE 2 Population Growth and Engineering Design

a. The population of Colorado was 3,827,000 in 1995 and 4,665,000 in 2005. Over this 10-year period, the average rate of change of the population was

$$\text{Rate of change} = \frac{\text{change in population}}{\text{change in years}}$$
$$= \frac{4,665,000 - 3,827,000}{2005 - 1995}$$
$$= 83,800 \text{ people per year.}$$

If Colorado's population continues to increase at this same rate for the next 10 years, it will have a 2015 population of 5,503,000 (see Figure 1.16). *(Source: U.S. Census Bureau)*

b. In tournament water-ski jumping, the ramp rises to a height of 6 feet on a raft that is 21 feet long, as shown in Figure 1.17. The slope of the ski ramp is the ratio of its height (the rise) to the length of its base (the run).
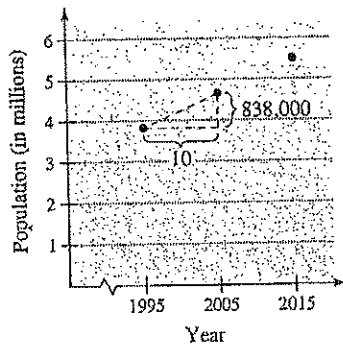
$$\text{Slope of ramp} = \frac{\text{rise}}{\text{run}} \qquad \text{Rise is vertical change, run is horizontal change.}$$
$$= \frac{6 \text{ feet}}{21 \text{ feet}}$$
$$= \frac{2}{7}$$

In this case, note that the slope is a ratio and has no units.

Population of Colorado
Figure 1.16
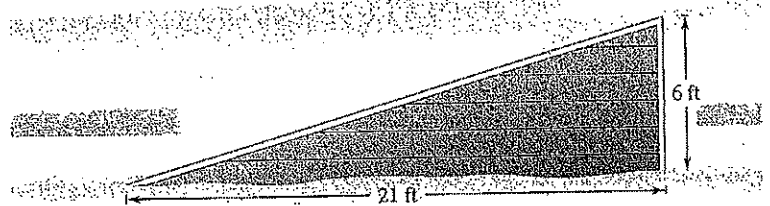
Dimensions of a water-ski ramp
Figure 1.17

The rate of change found in Example 2(a) is an **average rate of change.** A average rate of change is always calculated over an interval. In this case, the interv is [1995, 2005]. In Chapter 3 you will study another type of rate of change called a *instantaneous rate of change.*

6.2 NOTES

Because the slope of a vertical line is not defined, its equation cannot be written in the slope-intercept form. However, the equation of *any* line can be written in the **general form**

$$Ax + By + C = 0$$ General form of the equation of a line

where A and B are not *both* zero. For instance, the vertical line given by $x = a$ can be represented by the general form $x - a = 0$.

SUMMARY OF EQUATIONS OF LINES

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Point-slope form: $y - y_1 = m(x - x_1)$
5. Slope-intercept form: $y = mx + b$

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular, as shown in Figure 1.19. Specifically, nonvertical lines with the same slope are parallel and nonvertical lines whose slopes are negative reciprocals are perpendicular.
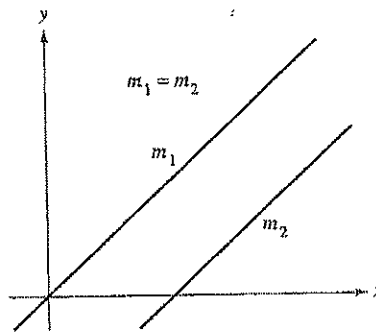
Parallel lines
Figure 1.19
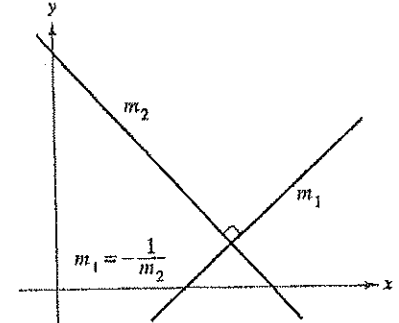
Perpendicular lines

STUDY TIP In mathematics, the phrase "if and only if" is a way of stating two implications in one statement. For instance, the first statement at the right could be rewritten as the following two implications.

a. If two distinct nonvertical lines are parallel, then their slopes are equal.

b. If two distinct nonvertical lines have equal slopes, then they are parallel.

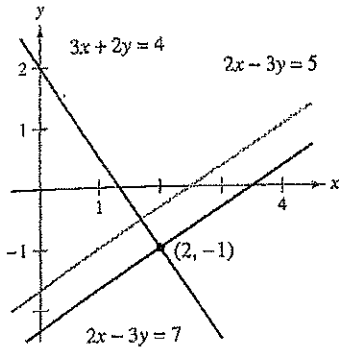**PARALLEL AND PERPENDICULAR LINES**

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal—that is, if and only if

   $$m_1 = m_2.$$

2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other—that is, if and only if

   $$m_1 = -\frac{1}{m_2}.$$

EXAMPLE 4   Finding Parallel and Perpendicular Lines

Find the general forms of the equations of the lines that pass through the point $(2, -1)$ and are

a. parallel to the line $2x - 3y = 5$.   **b.** perpendicular to the line $2x - 3y = 5$.

(See Figure 1.20.)

Solution   By writing the linear equation $2x - 3y = 5$ in slope-intercept form, $y = \frac{2}{3}x - \frac{5}{3}$, you can see that the given line has a slope of $m = \frac{2}{3}$.

a. The line through $(2, -1)$ that is parallel to the given line also has a slope of $\frac{2}{3}$.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
$$y - (-1) = \frac{2}{3}(x - 2) \qquad \text{Substitute.}$$
$$3(y + 1) = 2(x - 2) \qquad \text{Simplify.}$$
$$2x - 3y - 7 = 0 \qquad \text{General form}$$

Note the similarity to the original equation.

b. Using the negative reciprocal of the slope of the given line, you can determine that the slope of a line perpendicular to the given line is $-\frac{3}{2}$. So, the line through the point $(2, -1)$ that is perpendicular to the given line has the following equation.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
$$y - (-1) = -\frac{3}{2}(x - 2) \qquad \text{Substitute.}$$
$$2(y + 1) = -3(x - 2) \qquad \text{Simplify.}$$
$$3x + 2y - 4 = 0 \qquad \text{General form}$$

Lines parallel and perpendicular to
$2x - 3y = 5$
Figure 1.20

TECHNOLOGY PITFALL   The slope of a line will appear distorted if you use different tick-mark spacing on the x- and y-axes. For instance, the graphing calculator screens in Figures 1.21(a) and 1.21(b) both show the lines given by

$$y = 2x \quad \text{and} \quad y = -\frac{1}{2}x + 3.$$

Because these lines have slopes that are negative reciprocals, they must be perpendicular. In Figure 1.21(a), however, the lines don't appear to be perpendicular because the tick-mark spacing on the x-axis is not the same as that on the y-axis. In Figure 1.21(b), the lines appear perpendicular because the tick-mark spacing on the x-axis is the same as on the y-axis. This type of viewing window is said to have a *square setting*.
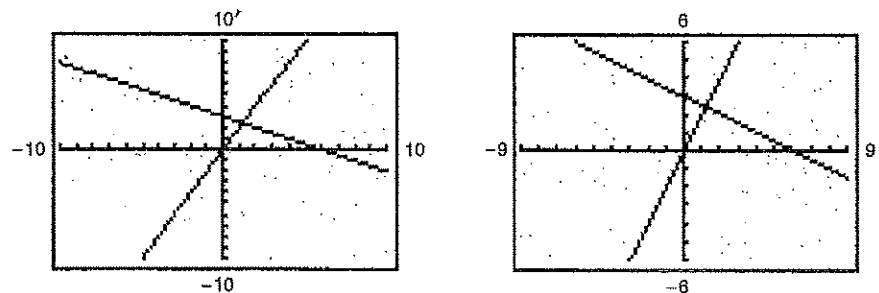
(a) Tick-mark spacing on the x-axis is not the same as tick-mark spacing on the y-axis.
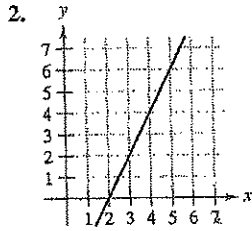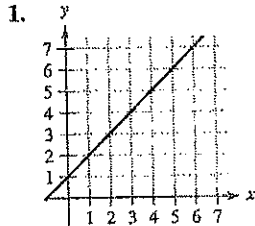
(b) Tick-mark spacing on the x-axis is the same as tick-mark spacing on the y-axis.

Figure 1.21

1.2 Exercises

See www.CalcChat.com for worked-out solutions to odd-numbered exercises.

In Exercises 1–6, estimate the slope of the line from its graph. To print an enlarged copy of the graph, go to the website www.mathgraphs.com.

1.

2.

3.

4.

5.

6.
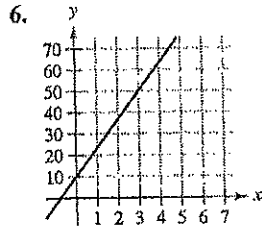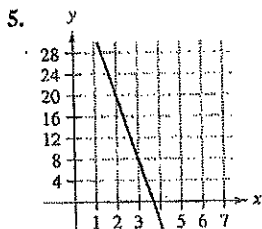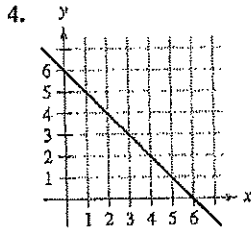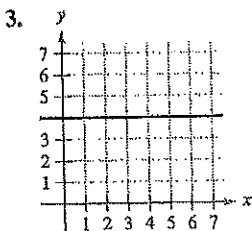
In Exercises 7 and 8, sketch the lines through the point with the given slopes. Make the sketches on the same set of coordinate axes.

Point	Slopes			
7. $(3, 4)$	(a) 1	(b) -2	(c) $-\frac{3}{2}$	(d) Undefined
8. $(-2, 5)$	(a) 3	(b) -3	(c) $\frac{1}{3}$	(d) 0

In Exercises 9–14, plot the pair of points and find the slope of the line passing through them.

9. $(3, -4), (5, 2)$

10. $(1, 1), (-2, 7)$

11. $(4, 6), (4, 1)$

12. $(3, -5), (5, -5)$

13. $\left(-\frac{1}{2}, \frac{2}{3}\right), \left(-\frac{3}{4}, \frac{1}{6}\right)$

14. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

In Exercises 15–18, use the point on the line and the slope of the line to find three additional points that the line passes through. (There is more than one correct answer.)

	Point	Slope		Point	Slope
15.	$(6, 2)$	$m = 0$	**16.**	$(-4, 3)$	m is undefined.
17.	$(1, 7)$	$m = -3$	**18.**	$(-2, -2)$	$m = 2$

19. *Conveyor Design*   A moving conveyor is built to rise 1 meter for each 3 meters of horizontal change.

(a) Find the slope of the conveyor.

(b) Suppose the conveyor runs between two floors in a factory. Find the length of the conveyor if the vertical distance between floors is 10 feet.

20. *Rate of Change*   Each of the following is the slope of a line representing daily revenue y in terms of time x in days. Use the slope to interpret any change in daily revenue for a one-day increase in time.

(a) $m = 800$ (b) $m = 250$ (c) $m = 0$

21. *Modeling Data*   The table shows the populations y (in millions) of the United States for 2000 through 2005. The variable t represents the time in years, with $t = 0$ corresponding to 2000. (*Source: U.S. Bureau of the Census*)

t	0	1	2	3	4	5
y	282.4	285.3	288.2	291.1	293.9	296.6

(a) Plot the data by hand and connect adjacent points with a line segment.

(b) Use the slope of each line segment to determine the year when the population increased least rapidly.

22. *Modeling Data*   The table shows the rate r (in miles per hour) that a vehicle is traveling after t seconds.

t	5	10	15	20	25	30
r	57	74	85	84	61	43

(a) Plot the data by hand and connect adjacent points with a line segment.

(b) Use the slope of each line segment to determine the interval when the vehicle's rate changed most rapidly. How did the rate change?

In Exercises 23–28, find the slope and the y-intercept (if possible) of the line.

23. $y = 4x - 3$

24. $-x + y = 1$

25. $x + 5y = 20$

26. $6x - 5y = 15$

27. $x = 4$

28. $y = -1$

In Exercises 29–34, find an equation of the line that passes through the point and has the given slope. Sketch the line.

	Point	Slope		Point	Slope
29.	$(0, 3)$	$m = \frac{3}{4}$	**30.**	$(-5, -2)$	m is undefined.
31.	$(0, 0)$	$m = \frac{2}{3}$	**32.**	$(0, 4)$	$m = 0$
33.	$(3, -2)$	$m = 3$	**34.**	$(-2, 4)$	$m = -\frac{3}{5}$

1.2 Exercises

In Exercises 35–44, find an equation of the line that passes through the points, and sketch the line.

35. $(0, 0), (4, 8)$

36. $(0, 0), (-1, 5)$

37. $(2, 1), (0, -3)$

38. $(-2, -2), (1, 7)$

39. $(2, 8), (5, 0)$

40. $(-3, 6), (1, 2)$

41. $(6, 3), (6, 8)$

42. $(1, -2), (3, -2)$

43. $\left(\frac{1}{2}, \frac{7}{2}\right), \left(0, \frac{3}{4}\right)$

44. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$

45. Find an equation of the vertical line with x-intercept at 3.

46. Show that the line with intercepts $(a, 0)$ and $(0, b)$ has the following equation.

$$\frac{x}{a} + \frac{y}{b} = 1, \quad a \neq 0, b \neq 0$$

In Exercises 47–50, use the result of Exercise 46 to write an equation of the line in general form.

47. x-intercept: $(2, 0)$
   y-intercept: $(0, 3)$

48. x-intercept: $\left(-\frac{2}{3}, 0\right)$
   y-intercept: $(0, -2)$

49. Point on line: $(1, 2)$
   x-intercept: $(a, 0)$
   y-intercept: $(0, a)$
   $(a \neq 0)$

50. Point on line: $(-3, 4)$
   x-intercept: $(a, 0)$
   y-intercept: $(0, a)$
   $(a \neq 0)$

In Exercises 51–58, sketch a graph of the equation.

51. $y = -3$

52. $x = 4$

53. $y = -2x + 1$

54. $y = \frac{1}{3}x - 1$

55. $y - 2 = \frac{3}{2}(x - 1)$

56. $y - 1 = 3(x + 4)$

57. $2x - y - 3 = 0$

58. $x + 2y + 6 = 0$

59. *Square Setting* Use a graphing utility to graph the lines $y = 2x - 3$ and $y = -\frac{1}{2}x + 1$ in each viewing window. Compare the graphs. Do the lines appear perpendicular? Are the lines perpendicular? Explain.

(a)
```
Xmin = -5
Xmax = 5
Xscl = 1
Ymin = -5
Ymax = 5
Yscl = 1
```

(b)
```
Xmin = -6
Xmax = 6
Xscl = 1
Ymin = -4
Ymax = 4
Yscl = 1
```

CAPSTONE

60. A line is represented by the equation $ax + by = 4$.

(a) When is the line parallel to the x-axis?

(b) When is the line parallel to the y-axis?

(c) Give values for a and b such that the line has a slope of $\frac{5}{8}$.

(d) Give values for a and b such that the line is perpendicular to $y = \frac{2}{5}x + 3$.

(e) Give values for a and b such that the line coincides with the graph of $5x + 6y = 8$.

In Exercises 61–66, write the general forms of the equations of the lines through the point (a) parallel to the given line and (b) perpendicular to the given line.

	Point	Line		Point	Line
61.	$(-7, -2)$	$x = 1$	62.	$(-1, 0)$	$y = -3$
63.	$(2, 1)$	$4x - 2y = 3$	64.	$(-3, 2)$	$x + y = 7$
65.	$\left(\frac{3}{4}, \frac{7}{8}\right)$	$5x - 3y = 0$	66.	$(4, -5)$	$3x + 4y = 7$

Rate of Change In Exercises 67–70, you are given the dollar value of a product in 2008 *and* the rate at which the value of the product is expected to change during the next 5 years. Write a linear equation that gives the dollar value V of the product in terms of the year t. (Let $t = 0$ represent 2000.)

	2008 Value	Rate
67.	$1850	$250 increase per year
68.	$156	$4.50 increase per year
69.	$17,200	$1600 decrease per year
70.	$245,000	$5600 decrease per year

In Exercises 71 and 72, use a graphing utility to graph the parabolas and find their points of intersection. Find an equation of the line through the points of intersection and graph the line in the same viewing window.
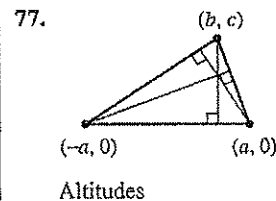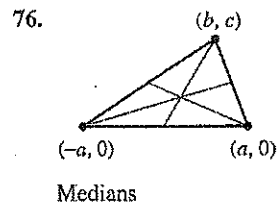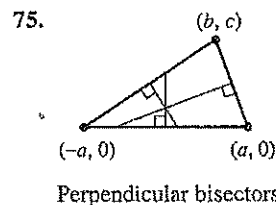
71. $y = x^2$
   $y = 4x - x^2$

72. $y = x^2 - 4x + 3$
   $y = -x^2 + 2x + 3$

In Exercises 73 and 74, determine whether the points are collinear. (Three points are *collinear* if they lie on the same line.)

73. $(-2, 1), (-1, 0), (2, -2)$

74. $(0, 4), (7, -6), (-5, 11)$

WRITING ABOUT CONCEPTS

In Exercises 75–77, find the coordinates of the point of intersection of the given segments. Explain your reasoning.
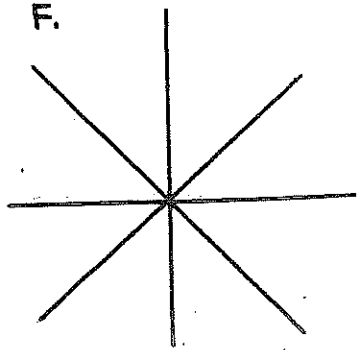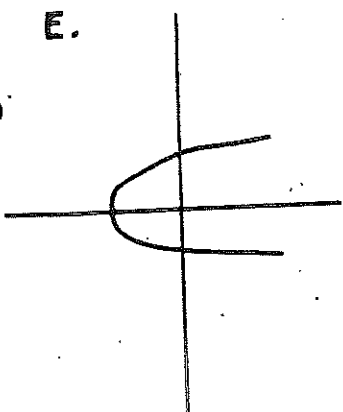
75.

(b, c)
$(-a, 0)$ $(a, 0)$
Perpendicular bisectors

76.

(b, c)
$(-a, 0)$ $(a, 0)$
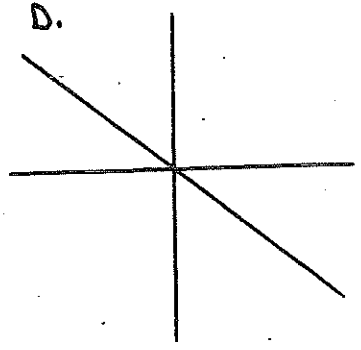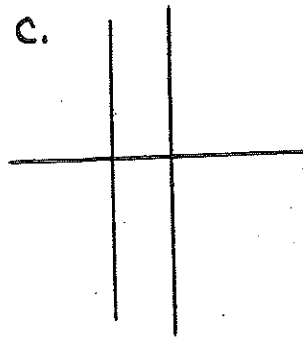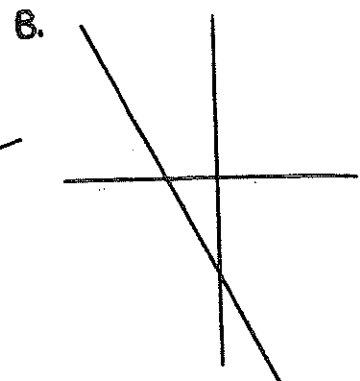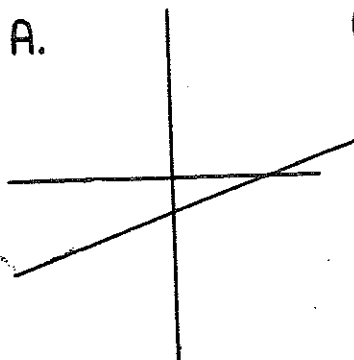Medians

77.

(b, c)
$(-a, 0)$ $(a, 0)$
Altitudes

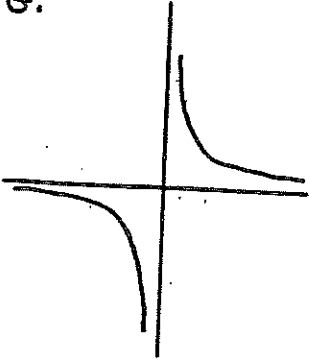78. Show that the points of intersection in Exercises 75, 76, and 77 are collinear.

Graphing Sprint

Identify which graph (by letter) fits best with the given equations. Place letter of the graph in space provided.

_____ 1. $7x + 2y + 11 = 0$

_____ 2. $y^2 - x - 1 = 0$

_____ 3. $y = 2x^2 + 4x - 1$

_____ 4. $y = \log_2 x$

_____ 5. $2x - 7y = 4$

_____ 6. $25x^2 - 9y^2 = 225$

_____ 7. $x + y = 0$

_____ 8. $y = 3x^4 - 4x^3 - 12x^2 + 12$

_____ 9. $x + 6 = 0$

_____ 10. $y = x^2 + 1$

_____ 11. $y = \dfrac{x^2}{x^2 - 9}$

_____ 12. $y = |x|$

_____ 13. $y = \sqrt{x - 3}$

_____ 14. $16x^2 - 25y^2 = 0$

_____ 15. $x^2 + y^2 - 10x + 6y + 18 = 0$

_____ 16. $y = 2^x$

_____ 17. $x^2 + y^2 = 49$

_____ 18. $y = |x + 1|$

_____ 19. $y = x^3$

_____ 20. $xy = 4$

A.

B.

C.

D.
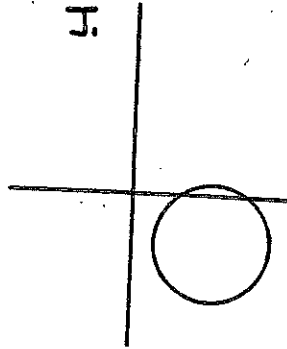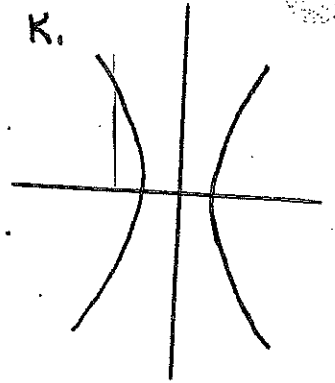
E.

F.

G.

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24

1.6 Notes
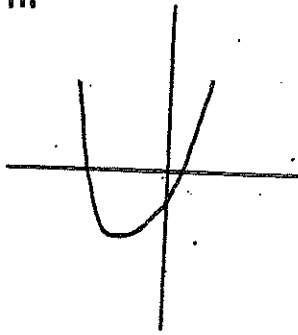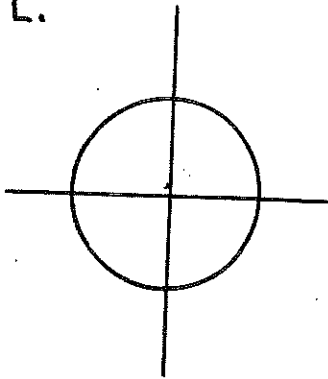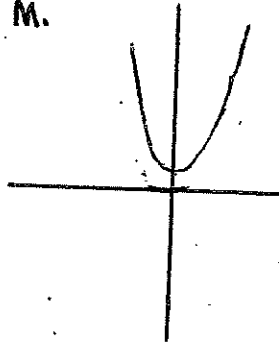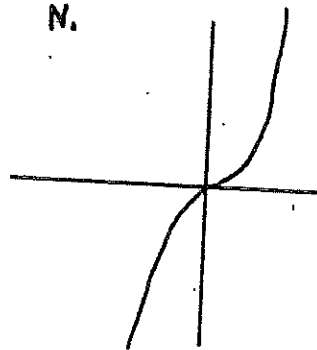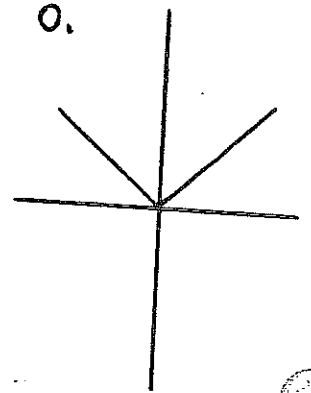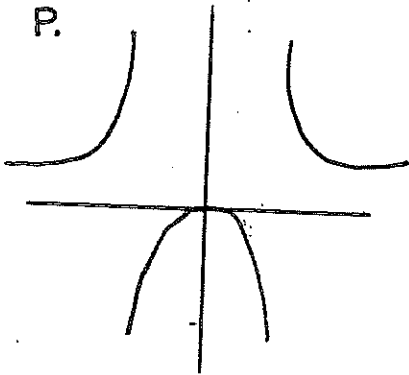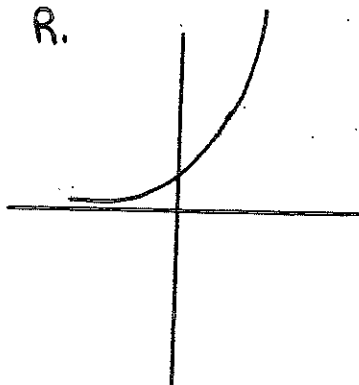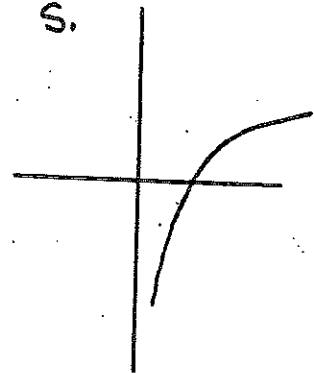
1.6 Exponential and Logarithmic Functions

■ Develop and use properties of exponential functions.
■ Understand the definition of the number e.
■ Understand the definition of the natural logarithmic function.
■ Develop and use properties of the natural logarithmic function.

Exponential Functions

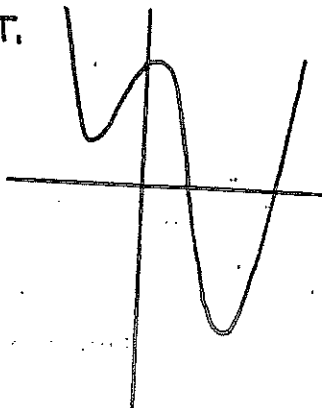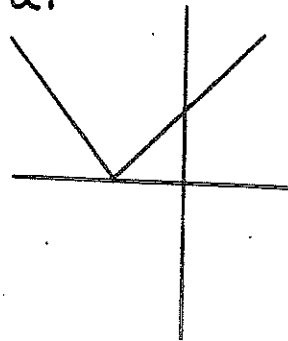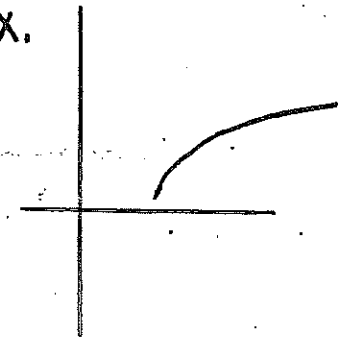An **exponential function** involves a constant raised to a power, such as $f(x) = 2^x$. You already know how to evaluate 2^x for *rational* values of x. For instance,

$$2^0 = 1, \quad 2^2 = 4, \quad 2^{-1} = \frac{1}{2}, \quad \text{and} \quad 2^{1/2} = \sqrt{2} \approx 1.4142136.$$

For *irrational* values of x, you can define 2^x by considering a sequence of rational numbers that approach x. A full discussion of this process would not be appropriate here, but the general idea is as follows. Suppose you want to define the number $2^{\sqrt{2}}$. Because $\sqrt{2} = 1.414213\ldots$, you consider the following numbers (which are of the form 2^r, where r is rational).

$$2^1 = 2 < 2^{\sqrt{2}} < 4 = 2^2$$
$$2^{1.4} = 2.639015\ldots < 2^{\sqrt{2}} < 2.828427\ldots = 2^{1.5}$$
$$2^{1.41} = 2.657371\ldots < 2^{\sqrt{2}} < 2.675855\ldots = 2^{1.42}$$
$$2^{1.414} = 2.664749\ldots < 2^{\sqrt{2}} < 2.666597\ldots = 2^{1.415}$$
$$2^{1.4142} = 2.665119\ldots < 2^{\sqrt{2}} < 2.665303\ldots = 2^{1.4143}$$
$$2^{1.41421} = 2.665137\ldots < 2^{\sqrt{2}} < 2.665156\ldots = 2^{1.41422}$$
$$2^{1.414213} = 2.665143\ldots < 2^{\sqrt{2}} < 2.665144\ldots = 2^{1.414214}$$

From these calculations, it seems reasonable to conclude that

$$2^{\sqrt{2}} \approx 2.66514.$$

In practice, you can use a calculator to approximate numbers such as $2^{\sqrt{2}}$.

In general, you can use any positive base a, $a \neq 1$, to define an exponential function. So, the exponential function with base a is written as $f(x) = a^x$. Exponential functions, even those with irrational values of x, obey the familiar properties of exponents.

> **PROPERTIES OF EXPONENTS**
>
> Let a and b be positive real numbers, and let x and y be any real numbers.
>
> **1.** $a^0 = 1$ **2.** $a^x a^y = a^{x+y}$ **3.** $(a^x)^y = a^{xy}$ **4.** $(ab)^x = a^x b^x$
>
> **5.** $\dfrac{a^x}{a^y} = a^{x-y}$ **6.** $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$ **7.** $a^{-x} = \dfrac{1}{a^x}$

EXAMPLE 1   Using Properties of Exponents

a. $(2^2)(2^3) = 2^{2+3} = 2^5$ b. $\dfrac{2^2}{2^3} = 2^{2-3} = 2^{-1} = \dfrac{1}{2}$

c. $(3^x)^3 = 3^{3x}$ d. $\left(\dfrac{1}{3}\right)^{-x} = (3^{-1})^{-x} = 3^x$

Properties of Logarithms

One of the properties of exponents states that when you multiply two exponential functions (having the same base), you add their exponents. For instance,

$$e^x e^y = e^{x+y}.$$

The logarithmic version of this property states that the natural logarithm of the product of two numbers is equal to the sum of the natural logs of the numbers. That is,

$$\ln xy = \ln x + \ln y.$$

This property and the properties dealing with the natural log of a quotient and the natural log of a power are listed here.

PROPERTIES OF LOGARITHMS

Let x, y, and z be real numbers such that $x > 0$ and $y > 0$.

1. $\ln xy = \ln x + \ln y$

2. $\ln \dfrac{x}{y} = \ln x - \ln y$

3. $\ln x^z = z \ln x$

EXAMPLE 5 Expanding Logarithmic Expressions

a. $\ln \dfrac{10}{9} = \ln 10 - \ln 9$ Property 2

b. $\ln \sqrt{3x + 2} = \ln(3x + 2)^{1/2}$ Rewrite with rational exponent.

$\qquad = \dfrac{1}{2}\ln(3x + 2)$ Property 3

c. $\ln \dfrac{6x}{5} = \ln(6x) - \ln 5$ Property 2

$\qquad = \ln 6 + \ln x - \ln 5$ Property 1

d. $\ln \dfrac{(x^2 + 3)^2}{x\sqrt[3]{x^2 + 1}} = \ln(x^2 + 3)^2 - \ln\left(x\sqrt[3]{x^2 + 1}\right)$

$\qquad = 2\ln(x^2 + 3) - [\ln x + \ln(x^2 + 1)^{1/3}]$

$\qquad = 2\ln(x^2 + 3) - \ln x - \ln(x^2 + 1)^{1/3}$

$\qquad = 2\ln(x^2 + 3) - \ln x - \dfrac{1}{3}\ln(x^2 + 1)$

When using the properties of logarithms to rewrite logarithmic functions, you must check to see whether the domain of the rewritten function is the same as the domain of the original function. For instance, the domain of $f(x) = \ln x^2$ is all real numbers except $x = 0$, and the domain of $g(x) = 2 \ln x$ is all positive real numbers.

TECHNOLOGY Try using a graphing utility to compare the graphs of

$$f(x) = \ln x^2 \quad \text{and} \quad g(x) = 2 \ln x.$$

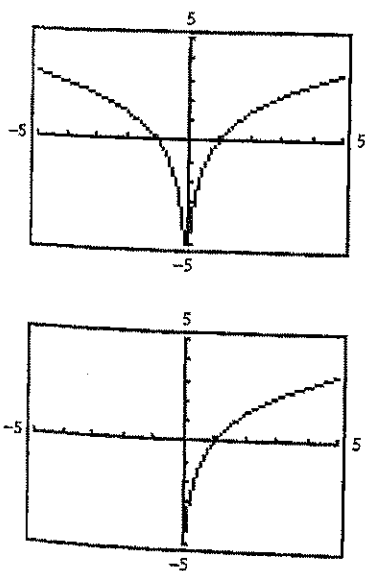Which of the graphs in Figure 1.52 is the graph of f? Which is the graph of g?

Figure 1.52

1.6 NOTES + EXERCISES

EXAMPLE 6 Solving Exponential and Logarithmic Equations

Solve (a) $7 = e^{x+1}$ and (b) $\ln(2x - 3) = 5$.

Solution

a.
$$7 = e^{x+1}$$ Write original equation.
$$\ln 7 = \ln(e^{x+1})$$ Take natural log of each side.
$$\ln 7 = x + 1$$ Apply inverse property.
$$-1 + \ln 7 = x$$ Solve for x.
$$0.946 \approx x$$ Use a calculator.

b.
$$\ln(2x - 3) = 5$$ Write original equation.
$$e^{\ln(2x-3)} = e^5$$ Exponentiate each side.
$$2x - 3 = e^5$$ Apply inverse property.
$$x = \frac{1}{2}(e^5 + 3)$$ Solve for x.
$$x \approx 75.707$$ Use a calculator.

1.6 Exercises

In Exercises 1 and 2, evaluate the expressions.

1. (a) $25^{3/2}$ (b) $81^{1/2}$ (c) 3^{-2} (d) $27^{-1/3}$

2. (a) $64^{1/3}$ (b) 5^{-4} (c) $\left(\frac{1}{8}\right)^{1/3}$ (d) $\left(\frac{1}{4}\right)^3$

In Exercises 3–6, use the properties of exponents to simplify the expressions.

3. (a) $(5^2)(5^3)$ (b) $(5^2)(5^{-3})$

   (c) $\dfrac{5^3}{25^2}$ (d) $\left(\dfrac{1}{4}\right)^2 2^6$

4. (a) $(2^2)^3$ (b) $(5^4)^{1/2}$

   (c) $[(27^{-1})(27^{2/3})]^3$ (d) $(25^{3/2})(3^2)$

5. (a) $e^2(e^4)$ (b) $(e^3)^4$

   (c) $(e^3)^{-2}$ (d) $\dfrac{e^5}{e^3}$

6. (a) $\left(\dfrac{1}{e}\right)^{-2}$ (b) $\left(\dfrac{e^5}{e^2}\right)^{-1}$

   (c) e^0 (d) $\dfrac{1}{e^{-3}}$

In Exercises 7–22, solve for x.

7. $3^x = 81$ 8. $4^x = 64$

9. $6^{x-2} = 36$ 10. $5^{x+1} = 125$

11. $\left(\frac{1}{2}\right)^x = 32$ 12. $\left(\frac{1}{4}\right)^x = 16$

13. $\left(\frac{1}{3}\right)^{x-1} = 27$ 14. $\left(\frac{1}{5}\right)^{2x} = 625$

15. $4^3 = (x + 2)^3$ 16. $18^2 = (5x - 7)^2$

17. $x^{3/4} = 8$ 18. $(x + 3)^{4/3} = 16$

19. $e^x = 5$ 20. $e^x = 1$

21. $e^{-2x} = e^5$ 22. $e^{3x} = e^{-4}$

OPP

In Exercises 23 and 24, compare the given number with the number e. Is the number less than or greater than e?

23. $\left(1 + \dfrac{1}{1,000,000}\right)^{1,000,000}$

24. $1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040}$

In Exercises 25–34, sketch the graph of the function.

25. $y = 3^x$ 26. $y = 3^{x-1}$

27. $y = \left(\frac{1}{3}\right)^x$ 28. $y = 2^{-x^2}$

29. $f(x) = 3^{-x^2}$ 30. $f(x) = 3^{|x|}$

31. $h(x) = e^{x-2}$ 32. $g(x) = -e^{x/2}$

33. $y = e^{-x^2}$ 34. $y = e^{-x/4}$

In Exercises 35–40, find the domain of the function.

35. $f(x) = \dfrac{1}{3 + e^x}$ 36. $f(x) = \dfrac{1}{2 - e^x}$

37. $f(x) = \sqrt{1 - 4^x}$ 38. $f(x) = \sqrt{1 + 3^{-x}}$

39. $f(x) = \sin e^{-x}$ 40. $f(x) = \cos e^{-x}$

41. Use a graphing utility to graph $f(x) = e^x$ and the given function in the same viewing window. How are the two graphs related?

   (a) $g(x) = e^{x-2}$ (b) $h(x) = -\frac{1}{2}e^x$ (c) $q(x) = e^{-x} + 3$

42. Use a graphing utility to graph the function. Describe the shape of the graph for very large and very small values of x.

   (a) $f(x) = \dfrac{8}{1 + e^{-0.5x}}$

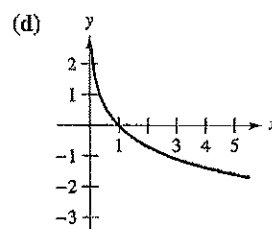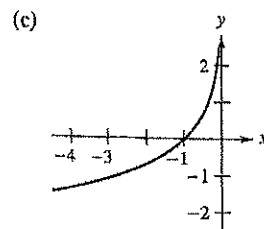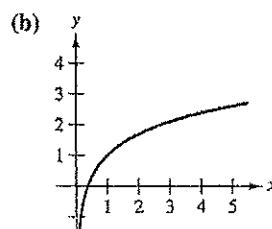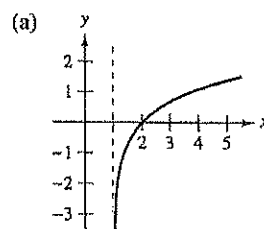   (b) $g(x) = \dfrac{8}{1 + e^{-0.5/x}}$

1.6 EXERCISES

In Exercises 43–46, match the equation with the correct graph. Assume that a and C are positive real numbers. [The graphs are labeled (a), (b), (c), and (d).]
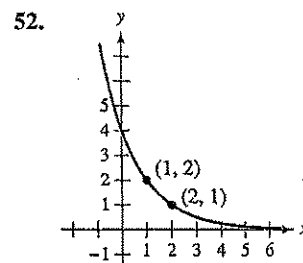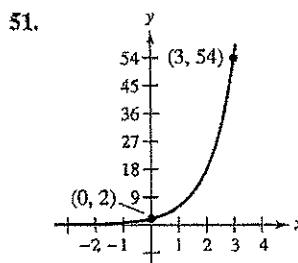
(a)

(b)

(c)

(d)
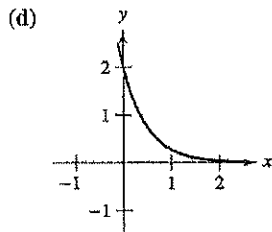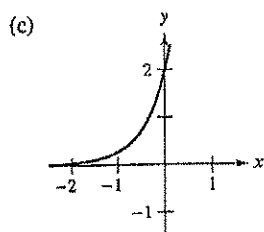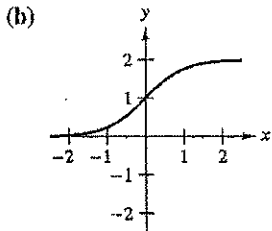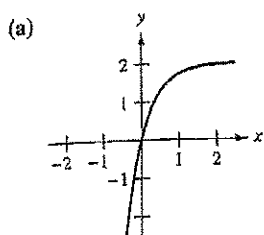
43. $y = Ce^{ax}$

44. $y = Ce^{-ax}$

45. $y = C(1 - e^{-ax})$

46. $y = \dfrac{C}{1 + e^{-ax}}$

In Exercises 47–50, match the function with its graph. [The graphs are labeled (a), (b), (c), and (d).]

(a)

(b)

(c)

(d)

47. $f(x) = \ln x + 1$

48. $f(x) = -\ln x$

49. $f(x) = \ln(x - 1)$

50. $f(x) = -\ln(-x)$

In Exercises 51 and 52, find the exponential function $y = Ca^x$ that fits the graph.

51.

52.

In Exercises 53–56, write the exponential equation as a logarithmic equation, or vice versa.

53. $e^0 = 1$

54. $e^{-2} = 0.1353\ldots$

55. $\ln 2 = 0.6931\ldots$

56. $\ln 0.5 = -0.6931\ldots$

In Exercises 57–62, sketch the graph of the function and state its domain.

57. $f(x) = 3 \ln x$

58. $f(x) = -2 \ln x$

59. $f(x) = \ln 2x$

60. $f(x) = \ln|x|$

61. $f(x) = \ln(x - 1)$

62. $f(x) = 2 + \ln x$

In Exercises 63–66, write an equation for the function having the given characteristics.

63. The shape of $f(x) = e^x$, but shifted eight units upward and reflected in the x-axis

64. The shape of $f(x) = e^x$, but shifted two units to the left and six units downward

65. The shape of $f(x) = \ln x$, but shifted five units to the right and one unit downward

66. The shape of $f(x) = \ln x$, but shifted three units upward and reflected in the y-axis

In Exercises 67–70, show that the functions f and g are inverses of each other by graphing them in the same viewing window.

67. $f(x) = e^{2x}$, $g(x) = \ln\sqrt{x}$

68. $f(x) = e^{x/3}$, $g(x) = \ln x^3$

69. $f(x) = e^x - 1$, $g(x) = \ln(x + 1)$

70. $f(x) = e^{x-1}$, $g(x) = 1 + \ln x$

In Exercises 71–74, (a) find the inverse of the function, (b) use a graphing utility to graph f and f^{-1} in the same viewing window, and (c) verify that $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$.

71. $f(x) = e^{4x-1}$

72. $f(x) = 3e^{-x}$

73. $f(x) = 2 \ln(x - 1)$

74. $f(x) = 3 + \ln(2x)$

In Exercises 75–80, apply the inverse properties of $\ln x$ and e^x to simplify the given expression.

75. $\ln e^{x^2}$

76. $\ln e^{2x-1}$

77. $e^{\ln(5x+2)}$

78. $e^{\ln\sqrt{x}}$

79. $-1 + \ln e^{2x}$

80. $-8 + e^{\ln x^3}$

In Exercises 81 and 82, use the properties of logarithms to approximate the indicated logarithms, given that $\ln 2 \approx 0.6931$ and $\ln 3 \approx 1.0986$.

81. (a) $\ln 6$ (b) $\ln \frac{2}{3}$ (c) $\ln 81$ (d) $\ln \sqrt{3}$

82. (a) $\ln 0.25$ (b) $\ln 24$ (c) $\ln \sqrt[3]{12}$ (d) $\ln \frac{1}{72}$

WRITING ABOUT CONCEPTS

83. In your own words, state the properties of the natural logarithmic function.

84. Explain why $\ln e^x = x$.

85. In your own words, state the properties of the natural exponential function.

86. The table of values below was obtained by evaluating a function. Determine which of the statements may be true and which must be false, and explain why.

(a) y is an exponential function of x.

(b) y is a logarithmic function of x.

(c) x is an exponential function of y.

(d) y is a linear function of x.

x	1	2	8
y	0	1	3

In Exercises 87–96, use the properties of logarithms to expand the logarithmic expression.

87. $\ln \dfrac{x}{4}$

88. $\ln \sqrt{x^5}$

89. $\ln \dfrac{xy}{z}$

90. $\ln(xyz)$

91. $\ln\left(x\sqrt{x^2 + 5}\right)$

92. $\ln \sqrt[3]{z + 1}$

93. $\ln \sqrt{\dfrac{x - 1}{x}}$

94. $\ln z(z - 1)^2$

95. $\ln (3e^2)$

96. $\ln \dfrac{1}{e}$

In Exercises 97–104, write the expression as the logarithm of a single quantity.

97. $\ln x + \ln 7$

98. $\ln y + \ln x^2$

99. $\ln(x - 2) - \ln(x + 2)$

100. $3 \ln x + 2 \ln y - 4 \ln z$

101. $\frac{1}{3}[2 \ln(x + 3) + \ln x - \ln(x^2 - 1)]$

102. $2[\ln x - \ln(x + 1) - \ln(x - 1)]$

103. $2 \ln 3 - \frac{1}{2}\ln(x^2 + 1)$

104. $\frac{3}{2}[\ln(x^2 + 1) - \ln(x + 1) - \ln(x - 1)]$

In Exercises 105–108, solve for x accurate to three decimal places.

105. (a) $e^{\ln x} = 4$

(b) $\ln e^{2x} = 3$

106. (a) $e^{\ln 2x} = 12$

(b) $\ln e^{-x} = 0$

107. (a) $\ln x = 2$

(b) $e^x = 4$

108. (a) $\ln x^2 = 8$

(b) $e^{-2x} = 5$

In Exercises 109–112, solve the inequality for x.

109. $e^x > 5$

110. $e^{1-x} < 6$

111. $-2 < \ln x < 0$

112. $1 < \ln x < 100$

In Exercises 113 and 114, show that $f = g$ by using a graphing utility to graph f and g in the same viewing window. (Assume $x > 0$.)

113. $f(x) = \ln(x^2/4)$

$g(x) = 2 \ln x - \ln 4$

114. $f(x) = \ln\sqrt{x(x^2 + 1)}$

$g(x) = \frac{1}{2}[\ln x + \ln(x^2 + 1)]$

115. Prove that $\ln (x/y) = \ln x - \ln y$, $x > 0, y > 0$.

116. Prove that $\ln x^y = y \ln x$.

117. Graph the functions

$f(x) = 6^x$ and $g(x) = x^6$

in the same viewing window. Where do these graphs intersect? As x increases, which function grows more rapidly?

118. Graph the functions

$f(x) = \ln x$ and $g(x) = x^{1/4}$

in the same viewing window. Where do these graphs intersect? As x increases, which function grows more rapidly?

119. Let $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$.

(a) Use a graphing utility to graph f and determine its domain.

(b) Show that f is an odd function.

(c) Find the inverse function of f.

CAPSTONE

120. Describe the relationship between the graphs of $f(x) = \ln x$ and $g(x) = e^x$.

Solving Review

Name _____

Find ALL solutions. Remember to check for extraneous solutions.
Show all work on another sheet of paper.

1. $\sqrt{3x+4}=\sqrt{x+18}$ _____

2. $x^2=3x+40$ _____

3. $2^{3x-1}=\dfrac{1}{32}$ _____

4. $\dfrac{x}{x-1}=\dfrac{6}{5}$ _____

5. $6x^4=54x^2$ _____

6. $3x^{\frac{1}{3}}+2x^{\frac{2}{3}}=5$ _____

7. $\dfrac{2}{x}=\dfrac{3}{x+2}-1$ _____

8. $\log_4\left(x+3\right)=\log_4\left(3x-7\right)$

_____

9. $3\sec^2 x=4$ _____

10. $(x-3)^{\frac{5}{6}}=3125$ _____

11. $\sqrt{11x+67}-x=7$ _____

12. $\sin x+1=\cos x$ _____

13. $3^{x+2}=5^{2x+7}$ _____

14. $x^4-5x^2+4=0$ _____

15. $\sin 3x=1$ _____

16. $\sqrt{2x-1}-\sqrt{x-5}=3$ _____

17. $|x+3|=12-2x$ _____

18. $3-\cos x=\sin^2 x$ _____

19. $\dfrac{4x-5}{3-7x}=2$ _____

20. $6\left(\dfrac{x}{x+1}\right)^2+5\left(\dfrac{x}{x+1}\right)-6=0$ _____

21. $\log_2\left(3x+17\right)-\log_2\left(x+2\right)=3$
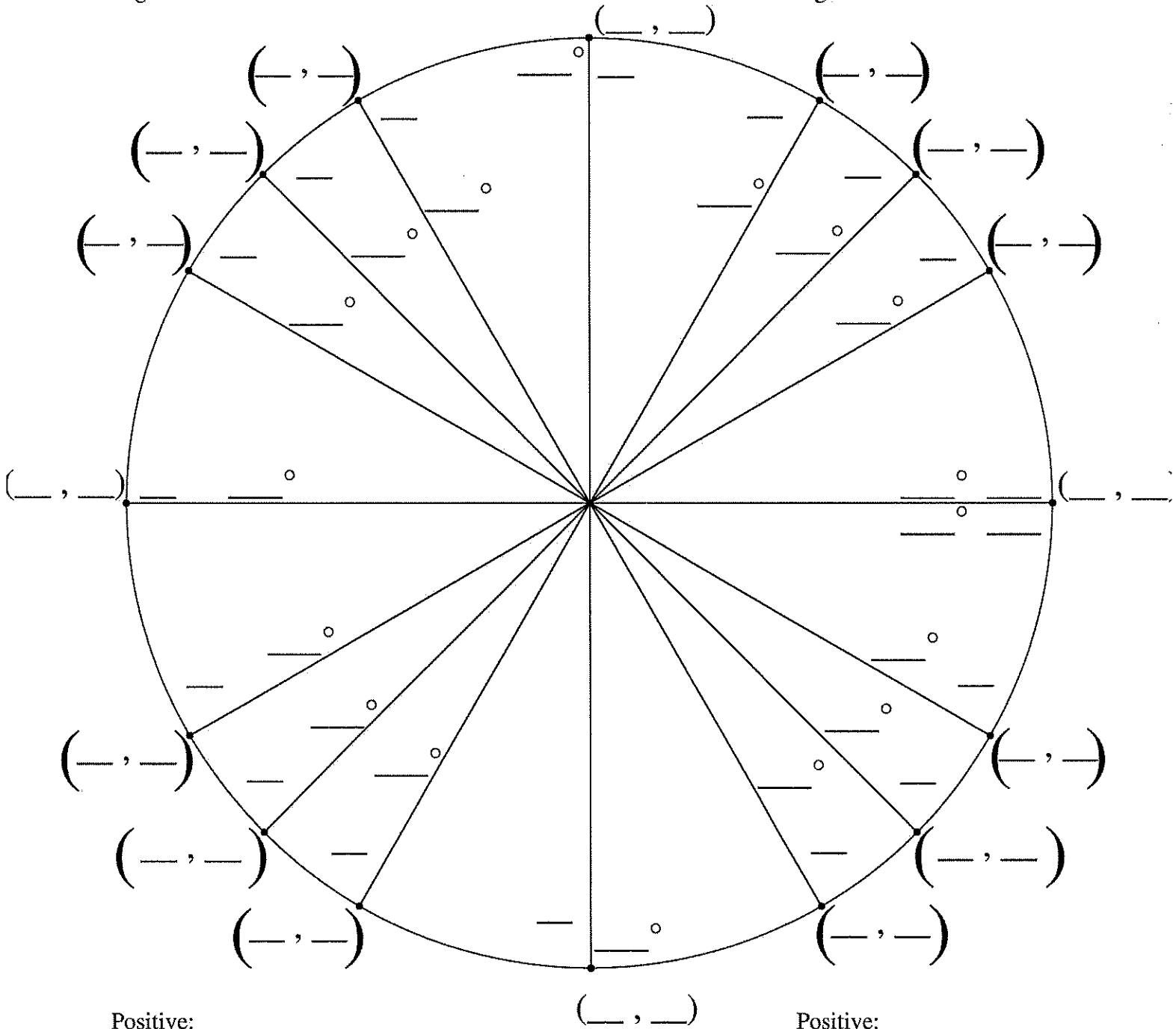
_____

22. $\cot^4 x+3\cot^2 x-4=0$

_____

23. $|x-4|=|-5|$ _____

24. $2x^5+5x^4-8x^3-20x^2=0$

_____

Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:

(__ , __)

(__ , __)

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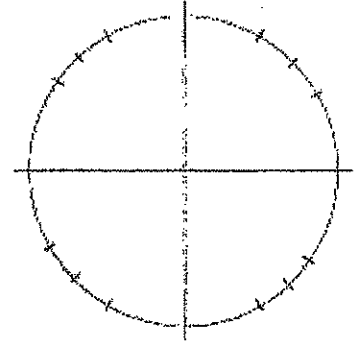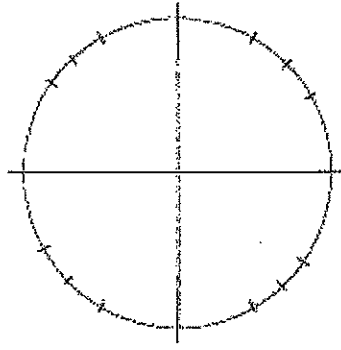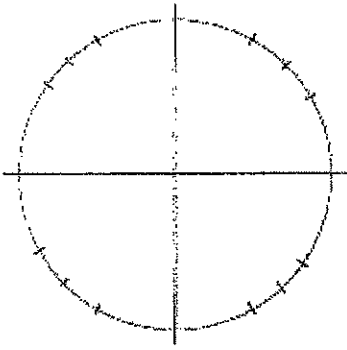
(__ , __)

(__ , __)

(__ , __)

Positive:
Negative:

Positive:
Negative:

Now fill in the chart using special right triangles. Recall All Students Take Classes. Use the circles below to help you.

θ	Radians	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0°							
30°							
45°							
60°							
90°							
120°							
135°							
150°							
180°							
210°							
225°							
240°							
270°							
300°							
315°							
330°							
360°							

Unit Circle Practice: Positive and Negative Radians and Degrees, all 6 Trig Functions

Name: _____   Date: _____   Period: _____

Part A: Evaluate each of the following (exact values, no calculators). Show work when appropriate (tan, cot, sec, csc):

1) $\sin 0$

2) $\sin -225°$

3) $\sin -210°$

4) $\sin -\dfrac{5\pi}{6}$

5) $\sin -45°$

6) $\cos 0$

7) $\sin 225°$

8) $\cos -210°$

9) $\sin 60°$

10) $\cos -225°$

11) $\cos 210°$

12) $\sin -\dfrac{11\pi}{6}$

13) $\sin 30°$

14) $\sin -30°$

15) $\sin \pi$

16) $\cos -\pi$

17) $\cos \dfrac{\pi}{6}$

18) $\cos -45°$

19) $\cos 45°$

20) $\sin \dfrac{11\pi}{6}$

21) $\sec -\dfrac{2\pi}{3}$

22) $\cot -\dfrac{\pi}{4}$

23) $\sec \dfrac{\pi}{3}$

24) $\cos -\dfrac{\pi}{2}$

25) $\cot 210°$

26) $\cot \dfrac{4\pi}{3}$

27) $\csc 30°$

28) $\sin -210°$

29) $\csc -\dfrac{3\pi}{2}$

30) $\tan -\dfrac{5\pi}{4}$

1) $\tan \dfrac{4\pi}{3}$

2) $\tan -\dfrac{\pi}{3}$

3) $\sin -\dfrac{\pi}{2}$

4) $\sin -\dfrac{3\pi}{2}$

5) $\cos \dfrac{\pi}{2}$

6) $\sin -\dfrac{4\pi}{3}$

7) $\cos 90°$

8) $\cos 270°$

9) $\cos -270°$

10) $\tan -300°$

11) $\tan -120°$

12) $\tan 300°$

13.) If there exists some angle, θ, where $\sec\theta = 5$, what is $\cos\theta$?

14.) If there exists some angle, θ, where $\tan\theta = \dfrac{-1}{3}$, what is $\cot\theta$?

15.) If there exists some angle, θ, where $\csc\theta = \dfrac{-5}{2}$, what is $\sin\theta$?

16.) $\sin 750° = $ _____

17.) $\cos -660° = $ _____

18.) $\tan \dfrac{11\pi}{4} = $ _____

19.) $\csc \dfrac{-17\pi}{6} = $ _____

20.) $\cot \dfrac{17\pi}{3} = $ _____

21.) $\sec \dfrac{-9\pi}{2} = $ _____

22.) $\csc -960° = $ _____

23.) $\cot 990° = $ _____