Standard Deviation & Normal Distribution Notes

Last new lesson of Algebra 2!

Yahoo!

Review: WHAT IS THE *MEAN* OF A SET OF DATA? the average

Standard Deviation is a statistical measure that shows how much data values deviate from the mean of a data set.

AKA – they tell us how spread out the data is!

For example, the <u>more spread out</u> the data is, the <u>larger</u> the standard deviation!

The formula for the ***standard deviation*** *is*: $\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

BUT…GOOD NEWS… the calculator will tell us the standard deviation if we enter in the data!!

Example 1: Below are the test scores of three students (Sally, Sue, Sandy).

Sally's scores: 70, 70, 70, 70, 70, 70 $\mu = 70$

Sue's scores: 75, 65, 73, 67, 71, 69 $\mu = 70$ **All three sets of data have the SAME*

Sandy's scores: 90, 50, 82, 58, 79, 61 $\mu = 70$ *MEAN (believe it or not)!*

Predict: a. Which of the students is going to have the highest standard deviation? Why?

Sandy scores are really spread out

b. What will the standard deviation for Sally's scores be? Why do you think so?

0, because none of the score deviate

sigma Σ

Now, calculate each σ by using your calculator:

Step 1: Stat → Edit → Enter data into L_1
Step 2: Stat → Calc → 1-Var Stats → Enter

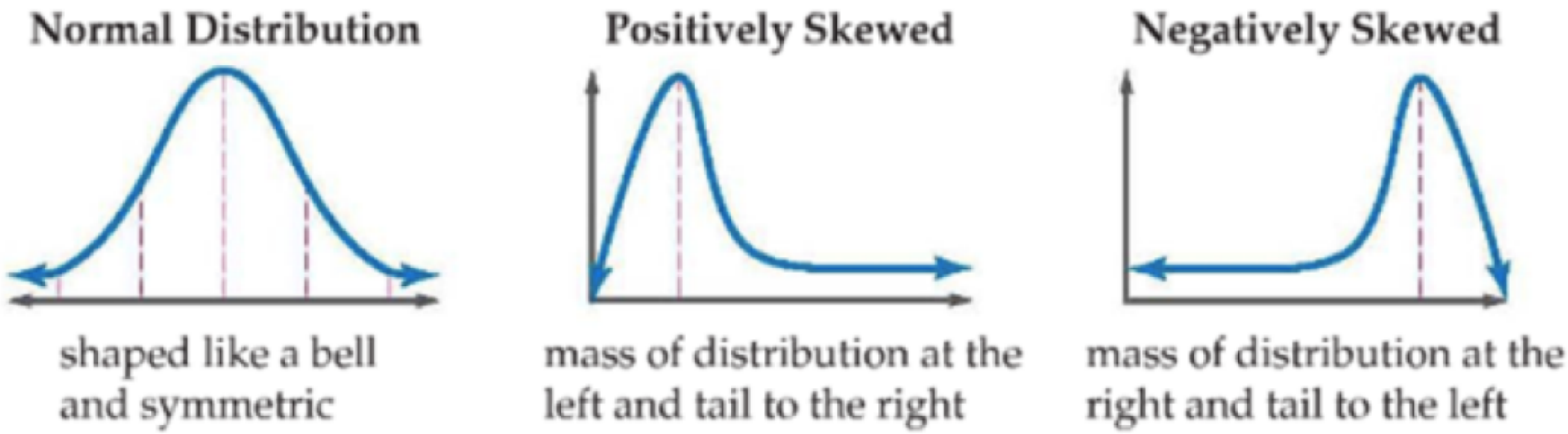
(mean: $\bar{x}$ standard deviation: σ)

Sally's σ 0

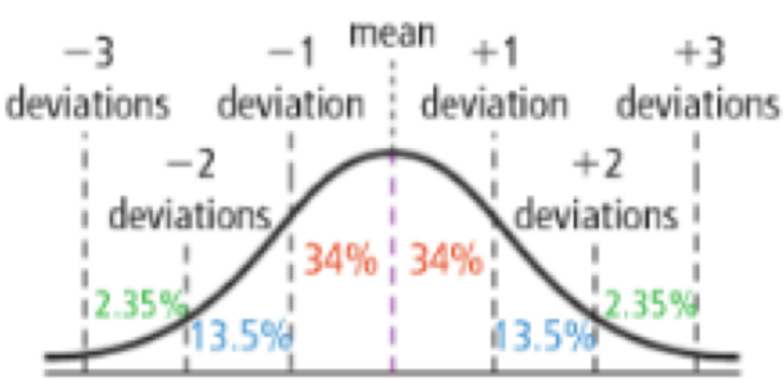
Sue's σ 3.42

Sandy's σ 14.3

THE NORMAL DISTRIBUTION



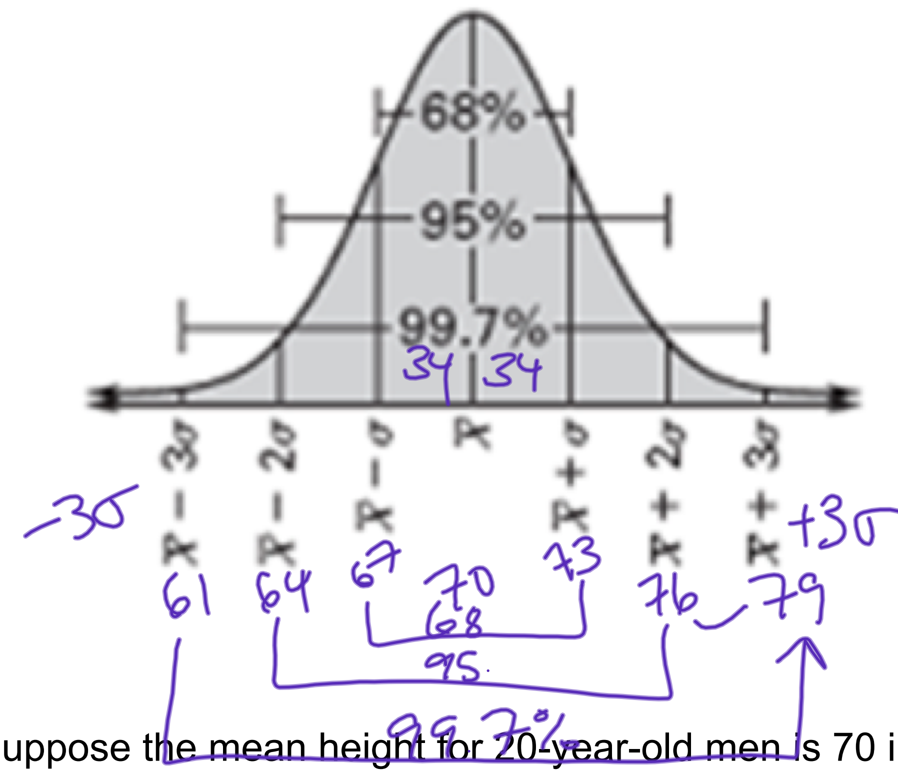
A normal distribution has data that vary randomly from the mean. The graph of a normal distribution is a normal curve.



In a normal distribution,

- 68% of data fall within one standard deviation of the mean
- 95% of data fall within two standard deviations of the mean
- 99.7% of data fall within three standard deviations of the mean

A normal distribution has a symmetric bell shape, centered at the mean. Many common statistics such as human height, weight or blood pressure have a normal distribution about the mean.

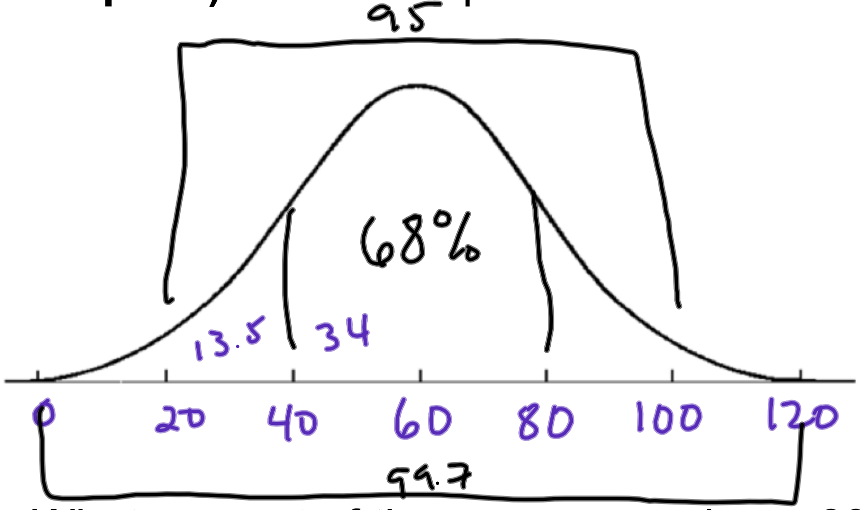


For example: Suppose the mean height for 20-year-old men is 70 inches and the standard deviation is 3 inches. This means that 68% of 20-year-old men have a height between 67 and 73 inches inclusive. Fill in the blanks below:

95% of 20-year-old men have a height between 64 and 76 inches inclusive.

99.7% of 20-year-old men have a height between 61 and 79 inches inclusive.

Example 4) Given the quiz scores 30 50 60 70 90. Draw a normal curve.



Mean = 60

Standard Deviation = 20

What percent of the scores are above 60? 50%

50 student

What percent of the scores are below 40? 16%

What percent of the scores are between 40 and 80? 68%

If 50 students took this quiz how many scored less than 40? $(.16)(50) = 8$

$\boxed{8 \text{ Students}}$

Z-Scores (standard deviations from mean)

A *z*-score reflects how many standard deviations above or below the mean a raw score is. The *z*-score is positive if the data value lies above the mean and negative if the data value lies below the mean.

raw score

mean

$$z = \frac{x - \mu}{\sigma}$$

std dev

Where x represents an element of the data set, the mean is represented by μ and standard deviation by σ.

Example 5) Suppose SAT scores among college students are normally distributed with a mean of 500 and a standard deviation of 100. If a student scores a 700, what would be her ***z***-score?

$$Z = \frac{700 - 500}{100} = \frac{200}{100} = 2$$

Her ***z***-score would be 2 which means her score is 2 standard deviations above the mean.

Example 6) In Harold's math class, a recent test has a mean of 70 and a standard deviation of 8. In Harold's English class, a recent test has a mean of 74 and a standard deviation of 16. If Harold earned a score of 78 on both tests, then in which subject is his performance better?

Find the z-score for each test:

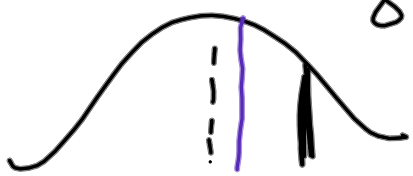
Math

$\mu = 70$

$\sigma = 8$

$x = 78$

$$z = \frac{78 - 70}{8} = \boxed{1}$$



English

$x = 78$

$\mu = 74$

$\sigma = 16$

$$z = \frac{78 - 74}{16} = \boxed{.25}$$

The Math score would have the highest standing since it is 1 standard deviation(s) above the mean, while the English score is only .25 standard deviation(s) above. the mean.