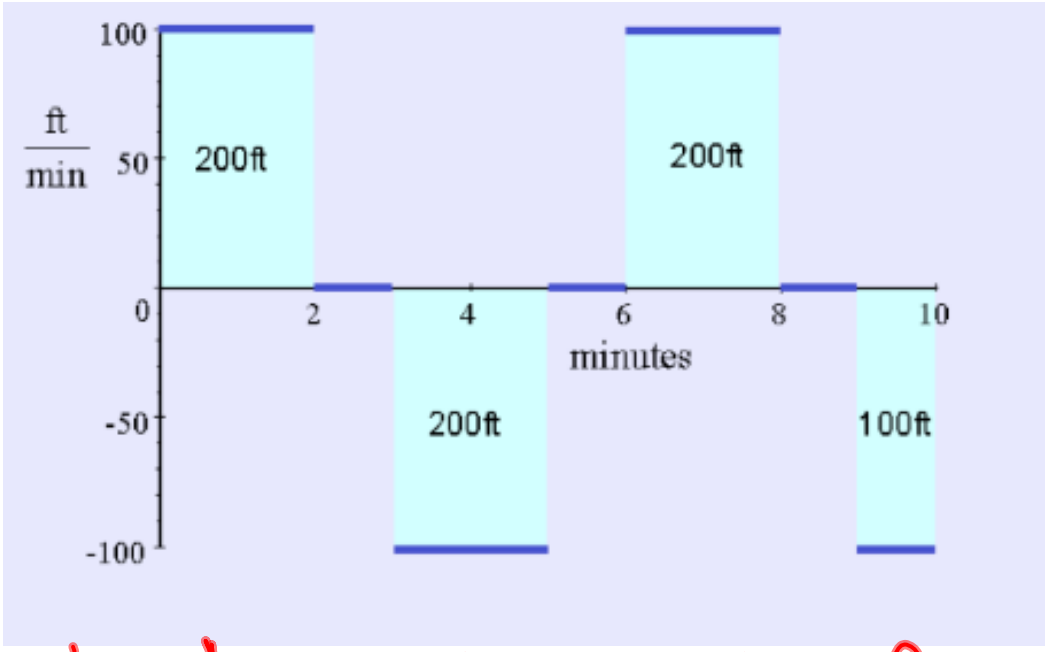
Distance, Displacement, and Position

Introduction: What is the difference between distance, displacement, and position?

Here's an example: A honey bee makes several trips from the hive to a flower garden. The velocity graph is shown below. What is the total distance traveled by the bee? What is the displacement of the bee? What is the position of the bee?



total distance = 700 ft.

displacement = 100 ft.

position = start pos + displacement = 100 ft.

Warm-up

A particle moves along the *x*-axis so that the acceleration at any time *t* is given by:

$$a(t) = 6t - 18$$

At time $t = 0$, the velocity of the particle is $v(0) = 24$ and at time $t = 1$, the position is $x(1) = 20$.

(a) Write an expression for the velocity $v(t)$ of the particle at any time t.

$v(t) = \int a(t)\,dt$

$v(t) = \int (6t - 18)\,dt$

$v(t) = \frac{6t^2}{2} - 18t + C$

$v(t) = 3t^2 - 18t + C$

$24 = 3(0)^2 - 18(0) + C$

$24 = C$

$\boxed{v(t) = 3t^2 - 18t + 24}$

(b) For what values of t is the particle at rest? when $v(t) = 0$

$3t^2 - 18t + 24 = 0$

$t^2 - 6t + 8 = 0$

$(t - 4)(t - 2) = 0$

$\boxed{t = 4,\ t = 2}$

(c) Write an expression for the position $x(t)$ of the particle at any time t.

$x(t) = \int v(t)\,dt$

$x(t) = \int (3t^2 - 18t + 24)\,dt$

$x(t) = \frac{3t^3}{3} - \frac{18t^2}{2} + 24t + C$

$x(t) = t^3 - 9t^2 + 24t + C$

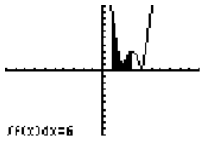
$20 = (1)^3 - 9(1)^2 + 24(1) + C$

$4 = C$

$\boxed{x(t) = t^3 - 9t^2 + 24t + 4}$

(d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

$\int_1^3 |v(t)|\,dt = \boxed{6}$



Warm-up Answers

(a) $v(t) = 3t^2 - 18t + 24$

(b) $t = 2, t = 4$

(c) $x(t) = t^3 - 9t^2 + 24t + 4$

(d) Total Distance $= 6$

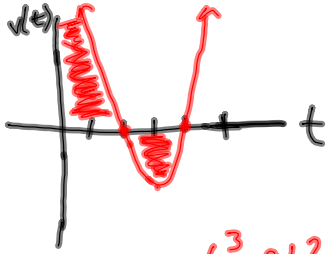
Now, using the equation from the warm-up find the following (WITHOUT A CALCULATOR):

(a) the total distance from 0 to 4

w/ calc: $\int_0^4 |v(t)|\,dt$

$v(t) = 3t^2 - 18t + 24$

Zeros: $t = 2, 4$



$$\int_0^2 v(t)\,dt + \left| \int_2^4 v(t)\,dt \right|$$

$$t^3 - 9t^2 + 24t \Big|_0^2 + \left| t^3 - 9t^2 + 24t \Big|_2^4 \right|$$

$$(20 - 0) + |-4|$$

$$20 + 4 = 24$$

(b) the displacement from 0 to 4

$$\int_0^4 v(t)\,dt = \left[t^3 - 9t^2 + 24t \right]_0^4$$

$$= 16 - 0$$

$$16$$

To find the displacement (position shift) from the velocity function, we just integrate the function. The negative areas below the *x*-axis subtract from the total displacement.

$$\text{Displacement} = \int_a^b v(t)\,dt$$

To find the distance traveled we have to use absolute value.

$$\text{Distance traveled} = \int_a^b |v(t)|\,dt$$

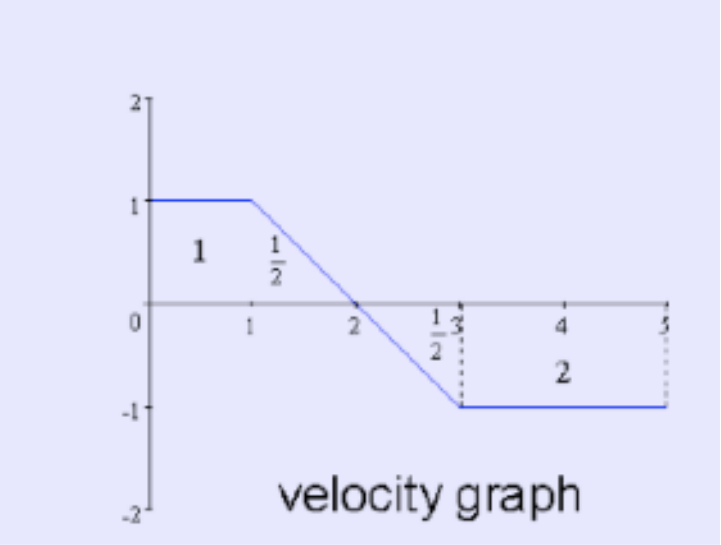
To find the distance traveled by hand you must:

Find the roots of the velocity equation and integrate in pieces, just like when we found the area between a curve and x-axis. (Take the absolute value of each integral.)

To find the distance traveled in your calculator you must:

Integrate the absolute value of the velocity function.

Example 1: Find the displacement and distance traveled using the velocity graph below.



Displacement:

$$1+\frac{1}{2}-\frac{1}{2}-2=-1$$

Distance Traveled:

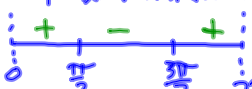
$$1+\frac{1}{2}+\frac{1}{2}+2=4$$

Example 2: Given $v(t) = 5\cos t, \quad 0 \le t \le 2\pi$

Analytically:

(a) Determine when the particle is moving to the right, to the left, and stopped.

1st Deriv Test (use v(t))



Stopped: $5\cos t = 0$, $\cos t = 0$, $t = \frac{\pi}{2}, \frac{3\pi}{2}$

moving right: $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$ b/c $v(t) > 0$

moving left: $(\frac{\pi}{2}, \frac{3\pi}{2})$ b/c $v(t) < 0$

(b) Find the particle's displacement for the given time interval.

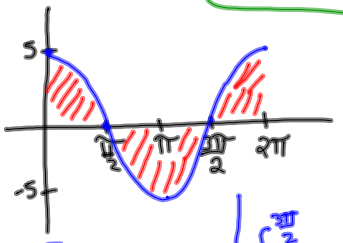
$$\int_0^{2\pi} 5\cos t\,dt = 5\sin t \Big|_0^{2\pi}$$

$$5\sin 2\pi - 5\sin 0$$

$$\boxed{0}$$

(c) Find the total distance traveled by the particle.

$$\int_0^{2\pi} |5\cos t|\,dt$$



$$\int_0^{\frac{\pi}{2}} 5\cos t\,dt + \left| \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 5\cos t\,dt \right| + \int_{\frac{3\pi}{2}}^{2\pi} 5\cos t\,dt$$

$$5\sin t \Big|_0^{\frac{\pi}{2}} + \left| 5\sin t \Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \right| + 5\sin t \Big|_{\frac{3\pi}{2}}^{2\pi}$$

$$(5\sin\tfrac{\pi}{2} - 5\sin 0) + \left|5\sin\tfrac{3\pi}{2} - 5\sin\tfrac{\pi}{2}\right| + (5\sin 2\pi - 5\sin\tfrac{3\pi}{2})$$

$$5 + |-5 - 5| + 0 - 5(-1)$$

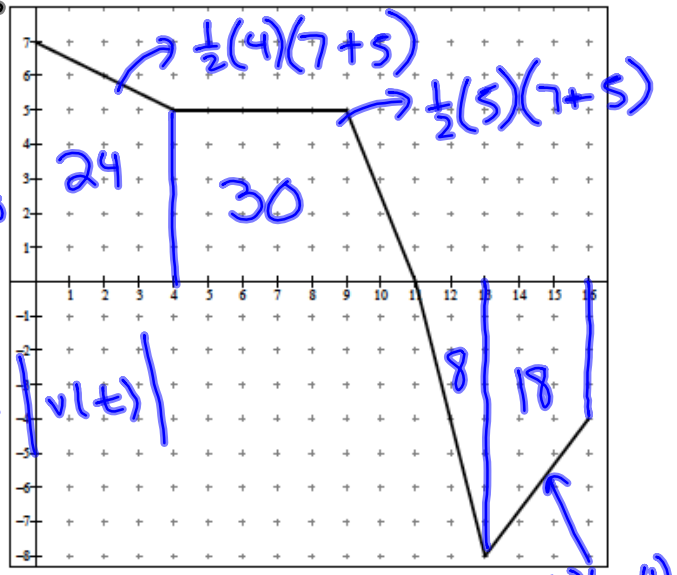
$$5 + |-10| + 5$$

$$\boxed{20}$$

→ start position

A particle moves along the x-axis and its position is given by the graph of the velocity function v ft/s

Given x(0) = -25 and the graph of v(t) for $0 \le t \le 16$



(a) Does the particle begin moving right or left?

right b/c v(t) > 0

(b) When is the particle at rest?

@ t = 11 secs b/c v(t) = 0

(c) What is the maximum velocity?

7 ft/sec @ t = 0

(d) What is the maximum speed of the particle?

8 ft/sec @ t = 13 speed = |v(t)|

(e) When is the particle moving to the left?

(11, 16) b/c v(t) < 0

(f) What is the total distance the particle travels?

$24 + 30 + 8 + 18 =$ **80 ft.**

(g) What is the x-coordinate of the particle's position when it is farthest to the right?

$-25 + \int_0^{11} v(t)\,dt = -25 + 54 =$ **29 ft.**

(h) What is the displacement of the particle for [0, 16]?

$\int_0^{16} v(t)\,dt = 24 + 30 - 8 - 18 =$ **28 ft.**

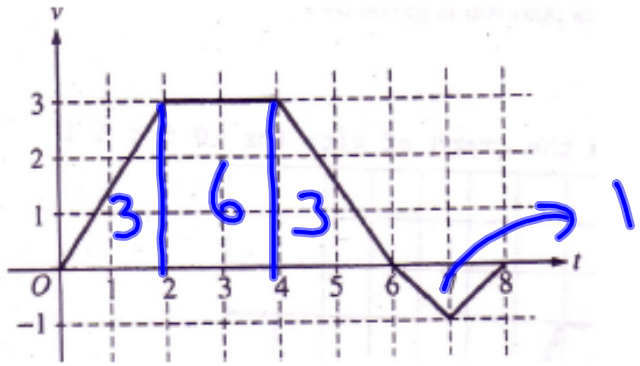
(i) What is the x-coordinate of the particle's finishing position?

x(t)

start pos + displacement

end pos $= -25 + \int_0^{16} v(t)\,dt = -25 + 28 =$ **3 ft.**

2. A bug begins to crawl up a vertical wire at time t = 0. The velocity v of the bug at time, t, where t is [0, 8], is given by the function whose graph is shown above.



i) At what value of t does the bug change direction?

(a) 2 (b) 4

(c) 6 (d) 7 (e) 8

ii) What is the total distance the bug traveled from t = 0 to t = 8?

(a) 14 (b) 13 (c) 11 (d) 8 (e) 6

3. An equation of the line tangent to the graph of $y = \cos(2x)$ at $x = \pi/4$ is

(a) $y - 1 = -\left(x - \pi/4\right)$

(b) $y - 1 = -2\left(x - \pi/4\right)$

(c) $y = 2\left(x - \pi/4\right)$

(d) $y = -\left(x - \pi/4\right)$

(e) $y = -2\left(x - \pi/4\right)$

$(\frac{\pi}{4}, 0)$

$\cos\left(2 \cdot \frac{\pi}{4}\right)$

$\cos\frac{\pi}{2} = 0$

$y' = -2\sin 2x$

$m_{tan}\left(\frac{\pi}{4}\right) = -2\sin\left(2 \cdot \frac{\pi}{4}\right)$

$-2\sin\frac{\pi}{2}$

$-2 \cdot 1 = -2$

$y - 0 = -2\left(x - \frac{\pi}{4}\right)$

4. An object moves along the x-axis with initial position x(0) = 2. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

(a) What is the acceleration of the object at time t = 4?

$a(t) = v'(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right)$ $a(4) = \frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right) = \frac{\pi}{3}\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6}}$

(b) Consider the following two statements.

Statement I: For 3< t < 4.5, the velocity of the object is decreasing.

$v(3) = \sin\pi = 0$ $v(4) = \sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$ → True

Statement II: For 3< t < 4.5, the speed of the object is increasing.

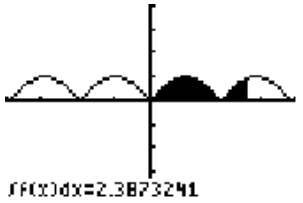
$|v(3)| = 0$ $|v(4)| = \frac{\sqrt{3}}{2}$ → True

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

Both are correct (See above work)

(c) What is the total distance traveled by the object over the time interval [0, 4]?

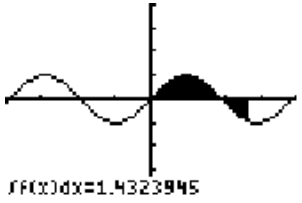
$\int_0^4 \left|\sin\frac{\pi}{3}t\right| dt$



$= \boxed{2.387}$

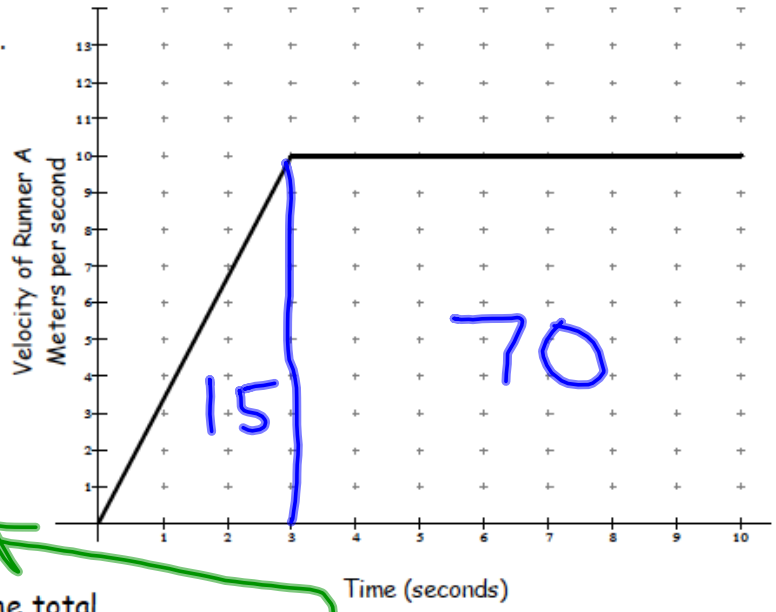
(d) What is the position of the object at time t = 4?

$2 + \int_0^4 \sin\frac{\pi}{3}t\, dt$



$2 + 1.432 = \boxed{3.432}$

5. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.



(a) Find the velocity of Runner A and the velocity of Runner B at time t = 2 seconds. Indicate units of measure.

A: 7 m/s B: $\frac{48}{7}$ m/s

(b) Find the acceleration of Runner A and the acceleration of Runner B at time t = 2 seconds. Indicate units of measure.

A: $\frac{10}{3}$ m/s² B: 1.47 m/s²

(c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

A: 85 m B: $\int_0^{10} |v(t)|\,dt$

B: 83.3 m

$a(t) = v'(t)$

$= \dfrac{24(2t+3) - 2(24t)}{(2t+3)^2}$

$a(t) = 1.47$ m/s²