

Algebra 2 – 8.1 Notes

Key Concept Direct Variation

Words y varies directly as x if there is some nonzero constant k such that $y = kx$. k is called the *constant of variation*.

Example If $y = 3x$ and $x = 7$, then $y = 3(7)$ or 21.

$$y = kx$$

Direct Variation Eq. vs. Non Direct Variation Eq.

$$y = 2x$$

$$y = -\frac{1}{2}x$$

$$y = \frac{3}{4}x$$

$$y = 2x - 1$$

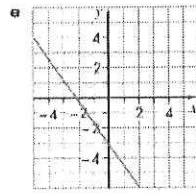
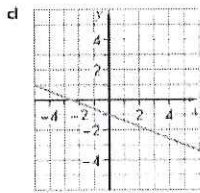
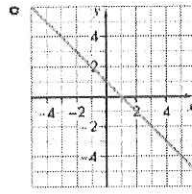
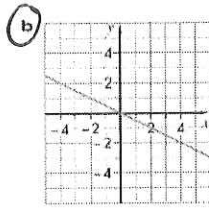
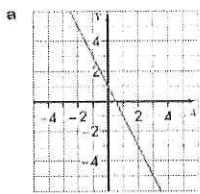
$$y = -\frac{1}{2}x + 2$$

$$y = \frac{3}{4}x + 5$$

What's the difference?

Direct Variation has a y -intercept of 0.

Which of the following represent direct variation?



Example 1: y varies directly with x . If $x = -2$ when $y = 8$, then what is x when $y = 18$?

$$y = kx$$

$$8 = k(-2)$$

$$k = -4$$

$$y = -4x$$

$$18 = -4x$$

$$x = -\frac{18}{4}$$

$$x = -\frac{9}{2}$$

Example 2: The weight of a plank varies directly with its length. If an 8 foot plank weighs 32 pounds, then what would a 12 foot plank weigh? Also, what's the constant of variation?

$$W = kL$$

$$32 = k(8)$$

$$k = 4$$

$$W = 4L$$

$$W = 4(12)$$

$$= 48 \text{ pounds}$$

$$k = 4$$

Example 3: y varies directly with x. If x = 25 when y = 5, then what is y when x = 8? What's the constant of variation?

$$y = kx$$

$$5 = k(25)$$

$$k = \frac{1}{5}$$

$$y = \frac{1}{5}x$$

$$y = \frac{1}{5}(8)$$

$$y = \frac{8}{5}$$

Example 4: The number of words Tom types varies directly with the amount of time he spends typing. If he types 300 words in 6 minutes, how long would it take him to type 525 words?

$$W = kT$$

$$300 = k(6)$$

$$k = 50$$

$$W = 50T$$

$$525 = 50T$$

$$T = 10.5 \text{ mins}$$

Which of the following tables represent direct variation?

(A)

x	4	7	-2	10
y	2	7/2	-1	5

B.

x	1	2	4	7
y	3	4	6	9

C.

x	2	3	4	5
y	10	15	21	25

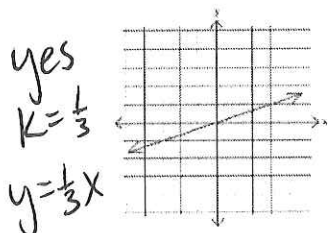
D.

x	1	2	7	27
y	3	6	21	9

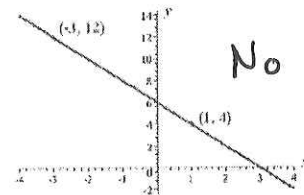
What is the constant of variation? $\frac{1}{2}$

What equation matches the table? $y = \frac{1}{2}x$

Determine whether the following represent direct variation and if so, name the constant of variation and write the equation.



$y = 2x - 1$
No



x	5	7	9	11
y	7	9	11	13

No

$D = 60t$
yes
 $k = 60$

x	-1	4	-8	5
y	4	-16	32	-20

yes
 $k = -4$
 $y = -4x$

Another type of variation is joint variation. **Joint variation** occurs when one quantity varies directly as the product of two or more other quantities.

KeyConcept Joint Variation

Words y varies jointly as x and z if there is some nonzero constant k such that $y = kxz$.

Example If $y = 5xz$, $x = 6$, and $z = -2$, then $y = 5(6)(-2)$ or -60 . $y = kxz$

Example 5: y varies jointly as x and z . If $y = 16$, when $x = 5$ and $z = 2$, find y when $x = 8$ and $z = 3$.

$$y = kxz$$

$$16 = k(5)(2)$$

$$16 = 10k$$

$$k = 1.6$$

$$y = 1.6xz$$

$$y = 1.6(8)(3)$$

$$y = 38.4$$

2 Inverse Variation and Combined Variation Another type of variation is inverse variation. If two quantities x and y show **inverse variation**, their product is equal to a constant k .

Inverse variation is often described as one quantity increasing while the other quantity is decreasing. For example, speed and time for a fixed distance vary inversely with each other; the faster you go, the less time it takes you to get there.

KeyConcept Inverse Variation

Words y varies inversely as x if there is some nonzero constant k such that $xy = k$ or $y = \frac{k}{x}$, where $x \neq 0$ and $y \neq 0$. $y = \frac{k}{x}$ or $xy = k$

Example If $xy = 2$, and $x = 6$, then $y = \frac{2}{6}$ or $\frac{1}{3}$.

Example 6: y varies inversely with x . If $y = 4$ when $x = -2$, what is x when $y = 7$?

$$y = \frac{k}{x}$$

$$4 = \frac{k}{-2}$$

$$k = -8$$

$$y = \frac{-8}{x}$$

$$7 = \frac{-8}{x}$$

$$7x = -8$$

$$x = -\frac{8}{7}$$

Example 7: The time it takes to travel a given distance varies inversely as the average rate of travel. Averaging 42 mph, it takes Andre 5 hours to drive to Houston. If it took him 4 hours and 20 minutes to reach Houston on his last trip, what was his average rate of travel?

$$T = \frac{k}{r}$$

$$5 = \frac{k}{42}$$

$$k = 210$$

$$T = \frac{210}{r}$$

$$4.\bar{3} = \frac{210}{r}$$

$$4.\bar{3}r = 210$$

$$r = 48 \text{ mph}$$

Example 8: The cost per person of renting a bus varies inversely with the number of people renting the bus. It costs \$33 per person if 26 people rent the bus. How much will it cost per person if 56 people rent the bus?

$$C = \frac{k}{p}$$

$$33 = \frac{k}{26}$$

$$k = 858$$

$$C = \frac{858}{p}$$

$$C = \frac{858}{56}$$

$$C = \$15.32 \text{ per person}$$

Combined Variation

Example 9: x varies directly with the square of y and inversely with the cube root of z. Write a model representing this.

$$x = \frac{ky^2}{\sqrt[3]{z}}$$

Example 10: Troy's likeability value, L, varies directly as the number of presents, P, he buys Gabriella and inversely as the square of the number of weeks, W, that they have been dating. After 3 weeks, he has purchased 2 presents and his likeability is 93. After 12 weeks, he has purchased only 3 presents. What is his likeability at this time?

$$L = \frac{kP}{w^2}$$

$$93 = \frac{k(2)}{(3)^2}$$

$$93 = \frac{2k}{9}$$

$$2k = 837$$

$$k = 418.5$$

$$L = \frac{418.5P}{w^2}$$

$$L = \frac{418.5(3)}{(12)^2}$$

$$L = 8.7$$