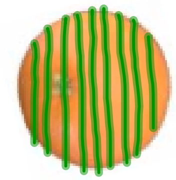
7.2 Volumes of Revolution Using the Disk Method

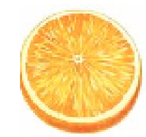
Intro:

How could we find the volume of an orange?



Slice it

If we sliced the orange, what shape would the slices be?



cylinders (flattened)

How could we use these slices to find the volume of the orange?

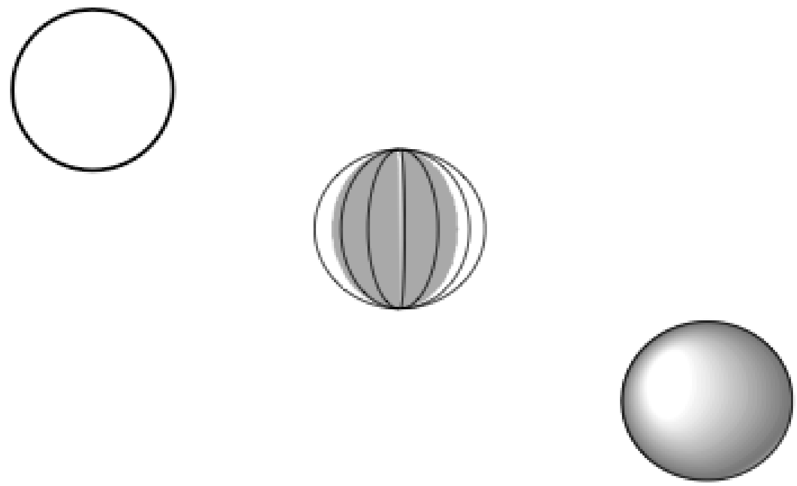
Add the volumes of <u>all</u> the cylinders. (disks)

How many slices should be use to get the most precise measurement?

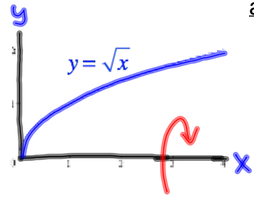
∞

What happens when a circle is rotated around its diameter?

it creates a sphere

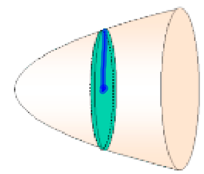


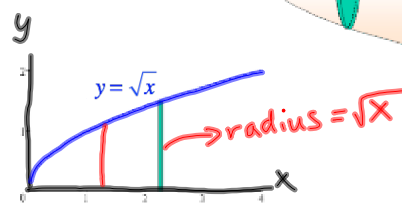
Volume of a Solid formed by rotation of a region around a horizontal or vertical line



How could we find the volume of the solid created by revolving this curve around the *x* - axis?

We could cut it into a series of thin slices or flat cylinders and add them together.





The volume of each flat cylinder or DISK is

$$\pi\left(\sqrt{x}\right)^2 dx$$

π r^2 height of disk (change in x)

If we add all the volumes together

$$\int_0^4 \pi\left(\sqrt{x}\right)^2 dx =$$

$$\pi\int_0^4 x\,dx$$

$$\pi \left.\frac{x^2}{2}\right|_0^4$$

$$= \pi\frac{16}{2} - 0 = \boxed{8\pi\ u^3}$$

- Calculus in Motion

http://www.ies-math.com/math/java/calc/rotate/rotate.html

This application of the method of slicing is called the disk method. The shape of the slice is a disk, so we use the formula for the volume of a cylinder to find the volume of the disk.

If the shape is rotated about the x-axis, then the formula is:

$$V = \pi \int_a^b y^2 \, dx \qquad \text{OR} \qquad V = \pi \int_a^b [f(x)]^2 \, dx$$

$\pi \cdot r^2 \cdot$ height of disk

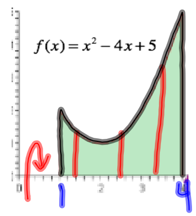
If the shape is rotated about the y-axis, then the formula is:

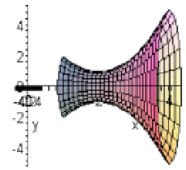
$$V = \pi \int_a^b \underbrace{[f(y)]^2}_{\text{or } x^2} \, dy$$

<u>Example 1:</u> Determine the volume of the solid obtained by rotating the region bounded by $f(x)=x^2-4x+5$, $x=1$, $x=4$, and the *x*-axis about the *x*-axis.

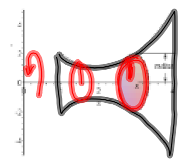
Remember, we are trying to add up an infinite number of slices.

1. Get a sketch of the bounding region and visualize the solid obtained by rotating the region about the x-axis.





2. Determine the volume of a slice.
The slice is perpendicular to the axis of rotation and is called a disk. Indicate a slice by drawing a representative slice through the region.



What is the radius of the slice?

$r = x^2-4x+5$
(the y-value)

Find the volume of the slice.

$\pi\left(x^2-4x+5\right)^2 dx$

3. Where do the slices start and stop?

$x=1$ $x=4$

4. Write the integral to find the volume of the solid of revolution and evaluate.

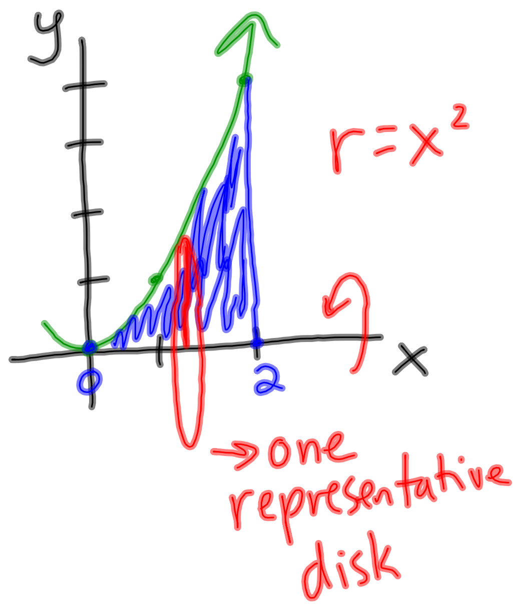
$$V=\pi\int_1^4\left(x^2-4x+5\right)^2dx$$

use calculator to find

$$=15.6\pi \approx \boxed{49u^3}$$

Example 2: Determine the volume of the solid formed by revolving the region bounded by the graph of $y = x^2$, the *x*-axis, and $0 \le x \le 2$ about the x-axis.

http://www.math.psu.edu/dlittle/java/calculus/volumedisks.html



$$\pi \int_0^2 (x^2)^2 \, dx$$

$$\pi \int_0^2 x^4 \, dx$$

$$\pi \left. \frac{x^5}{5} \right|_0^2$$

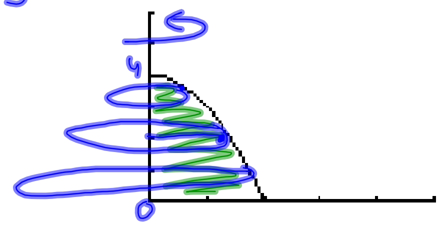
$$\boxed{\frac{32\pi}{5} u^3}$$

Example 3:

$y = 4 - x^2$ is bounded by the x-axis and the y-axis in the first quadrant and is revolved about the <u>y-axis</u>. Find the volume of the solid formed.

$\to x^2 = 4 - y \to x = \sqrt{4-y}$

1) A sketch of the region to be rotated is drawn to the right.
Indicate a representative slice and draw an arrow showing the rotation.



2) Identify the radius. (x-value)

$r = \sqrt{4-y}$

3) Write the expression for the volume of one disk.

$\pi\left(\sqrt{4-y}\right)^2 dy$

4) Determine where the slices start and stop.

$y=0 \quad y=4$

5) Write the integral and evaluate.

$$V = \pi \int_0^4 \left(\sqrt{4-y}\right)^2 dy$$

$$\pi \int_0^4 4-y \; dy$$

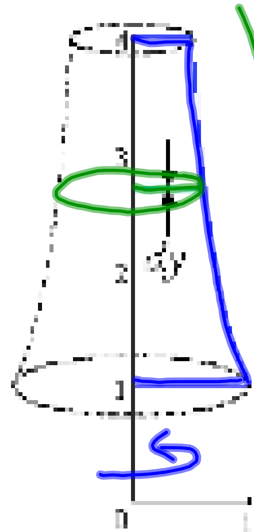
$$\pi \left[4y - \frac{y^2}{2}\right]_0^4$$

$$8\pi \; u^3$$

Example 4: The region between the curve $x = \dfrac{1}{\sqrt{y}}$, $1 \le y \le 4$ and the *y*-axis is revolved about the *y*-axis. Find the volume.

*To draw a graph, create an *x*-*y* chart by choosing *y*-values:

x	y
1	1
$\frac{1}{\sqrt{2}} \approx 0.707$	2
$\frac{1}{\sqrt{3}} \approx 0.577$	3
$\frac{1}{2}$	4



$$V = \pi \int_1^4 \left(\frac{1}{\sqrt{y}}\right)^2 dy$$

$$= \pi \int_1^4 \frac{1}{y}\, dy$$

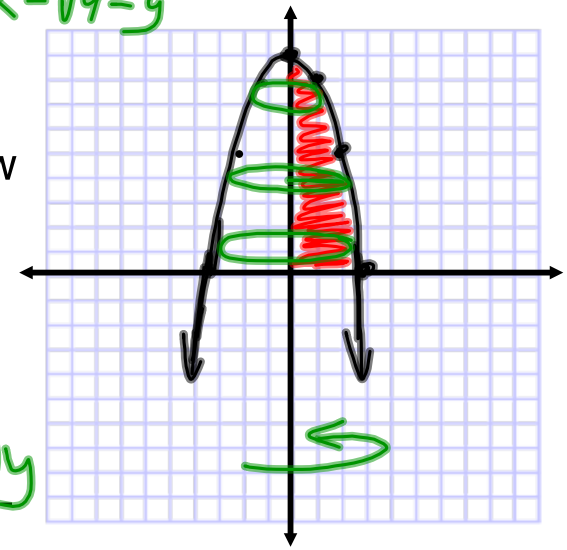
$$\pi \left[\ln y\right]_1^4$$

$$\pi\left[\ln 4 - \ln 1\right] = \pi \ln 4 \ u^3$$

Example 5: The region bounded by $y = 9 - x^2$, the *x*-axis, and the *y*-axis is rotated around the y-axis. Find the volume of the solid formed.

$x^2 = 9 - y \rightarrow x = \sqrt{9-y}$

Draw a sketch of the region to be rotated.



Indicate a representative slice and draw an arrow showing the rotation.

Identify the length of the radius: $\sqrt{9-y}$

Write an expression for Volume: $\pi\left(\sqrt{9-y}\right)^2 dy$

Determine where the slices start and stop: $y=0,\ y=9$

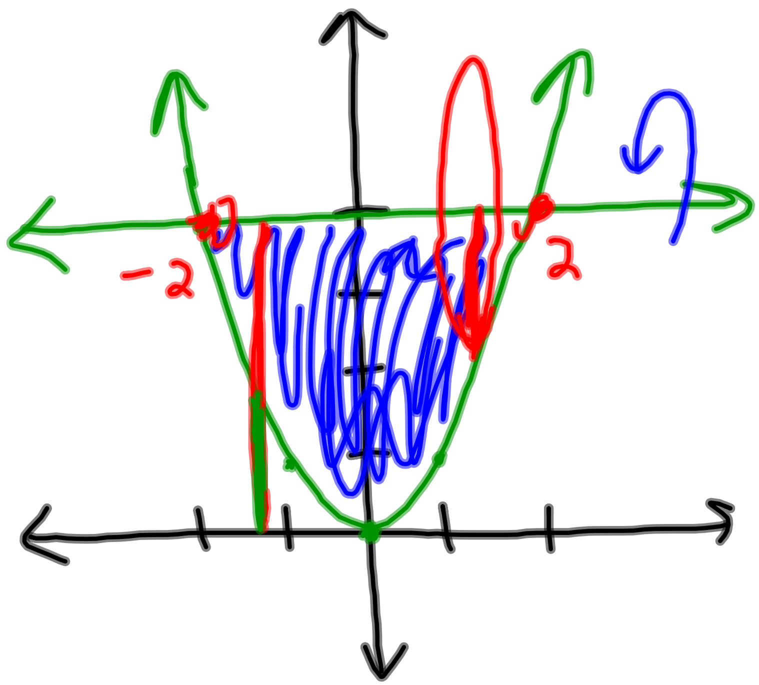
Write the integral and evaluate: $V = \pi\int_0^9 \left(\sqrt{9-y}\right)^2 dy$

$\frac{81\pi}{2} = 127.23u^3$

Example 6:

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line *y* = 4.

Equations: $y = x^2, \;\; y = 4$



r = top - bottom

$r = 4 - x^2$

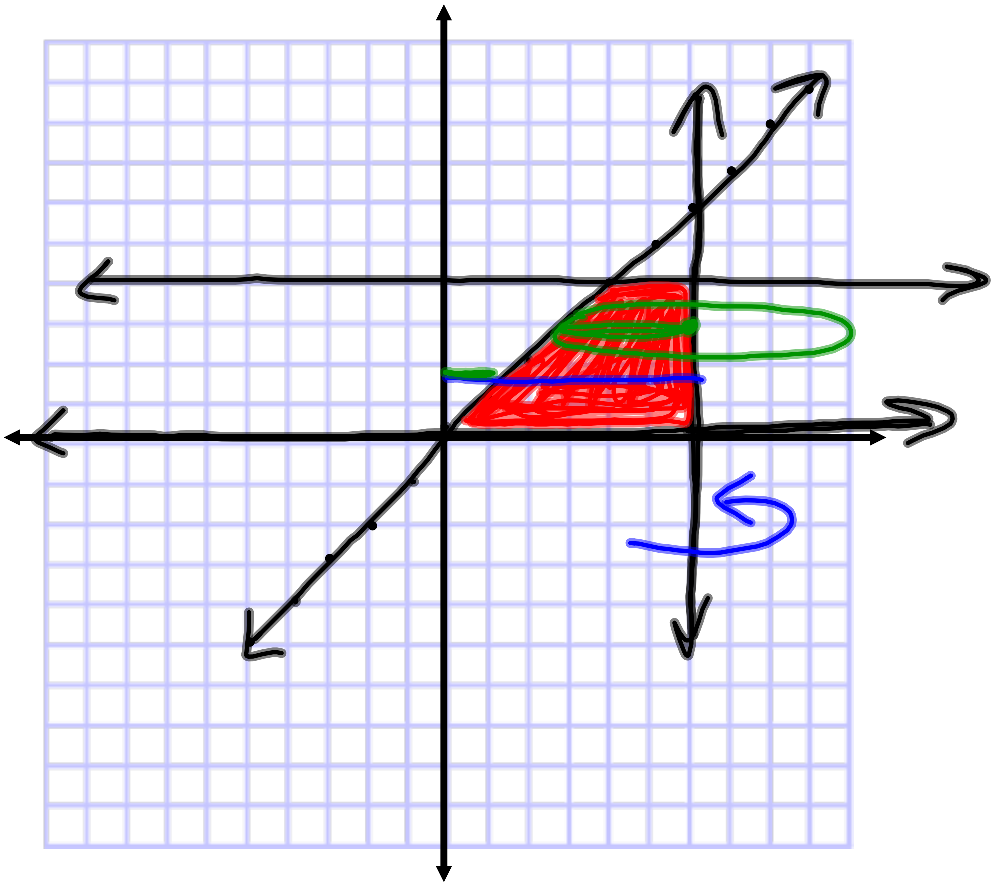
$$V = \pi \int_{-2}^{2} (4-x^2)^2 \, dx$$

$$V = 34.1\overline{3}\pi \approx \boxed{107.233 \; u^3}$$

Example 7:

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the line x = 6.

Equations: $y = x, \; y = 0, \; y = 4, \; x = 6$



r = right − left

$r = 6 - y$

$$V = \pi \int_0^4 (6-y)^2 \, dy$$

$$= \boxed{217.817 u^3}$$

http://www.calculusapplets.com/revolution.html

http://clem.mscd.edu/~talmanl/HTML/VolumeOfRevolution.html

http://clem.mscd.edu/~talmanl/HTML/VolumeOfRevolution02.html

http://clem.mscd.edu/~talmanl/HTML/DetailedVolRev.html

http://www.ies.co.jp/math/products/calc/applets/rotate/rotate.html