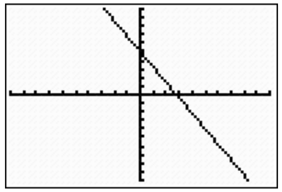
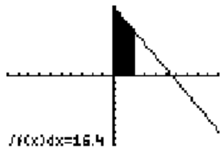
5.4 FUNDAMENTAL THEOREM OF CALCULUS



If a particle's velocity is displayed by the function y = -1.8t + 10, how far has the particle traveled in the first two seconds?

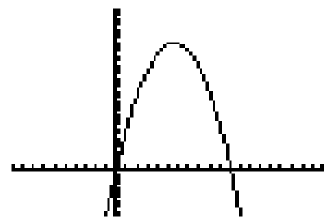
$$\int_0^2 v(t)\,dt = \text{Distance traveled in first 2 secs.}$$



$$\int_0^2 (-1.8t+10)\,dt = \boxed{16.4 \text{ units}}$$

What is a function for the particle's position? Assume its initial position is 0.

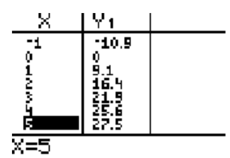
$$x(t) = -0.9t^2 + 10t$$



Could this function be used to evaluate how far the particle has traveled in the first two seconds? yes

$$x(2) - x(0) = 16.4 - 0 = \boxed{16.4 \text{ units}}$$

Are your answers the same? yes



Fundmental Theorem of Calculus

If a function *f* is continuous on the closed interval [*a*,*b*] and *F* is the antiderivative of *f* on the interval [*a*,*b*], then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Using the Fundamental Theorem of Calculus

1. Provided you can find an antiderivative of f, you now have a way to evaluate a definite integral without having to use Riemann Sums.

2. When applying the FTC, the following notation is convenient

$$\int_a^b f(x)dx = F(x)\Big|_a^b \quad \text{upper} - \text{lower}$$

$$= F(b) - F(a)$$

For example: $\int_1^3 x^3\,dx = \frac{x^4}{4}\Big|_1^3 = \frac{(3)^4}{4} - \frac{(1)^4}{4}$

$$\frac{81}{4} - \frac{1}{4} = \frac{80}{4}$$

$$= \boxed{20}$$

3. It is not necessary to include a constant of integration C in the antiderivative because

$$\int_a^b f(x)dx = [F(x)+C]_a^b$$

$$= [F(b)+C] - [F(a)+C]$$

$$= F(b) - F(a)$$

Fundamental Theorem of Calculus

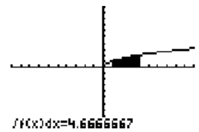
- establishes a link between the two branches of Calculus:
  - differential calculus and integral calculus

Part I of FTC states

1. every continuous function has an antiderivative
2. the processes of integration and differentiation are inverses of one another

Examples: Evaluate each integral. Be sure to use proper notation.

a. $\int_1^4 x^{\frac{1}{2}}\,dx = \dfrac{2x^{\frac{3}{2}}}{3}\Bigg|_1^4 = \dfrac{2(4)^{\frac{3}{2}}}{3} - \dfrac{2(1)^{\frac{3}{2}}}{3}$



$= \dfrac{2(8)}{3} - \dfrac{2(1)}{3}$

$\dfrac{16}{3} - \dfrac{2}{3} = \boxed{\dfrac{14}{3}}$

b. $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x\,dx = \sin x\Bigg|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin\frac{\pi}{6}$

$1 - \frac{1}{2}$

$\boxed{\frac{1}{2}}$

c. $\int_6^{10} 3\,dx = \Big[3x\Big]_6^{10} = 3(10) - 3(6)$

$30 - 18$

$\boxed{12}$

d. $\int_1^2 (x^2 - 3)\,dx = \left[\dfrac{x^3}{3} - 3x\right]_1^2$

$= \left(\dfrac{2^3}{3} - 3(2)\right) - \left(\dfrac{1^3}{3} - 3(1)\right)$

$= \left(\frac{8}{3} - 6\right) - \left(\frac{1}{3} - 3\right) = -\frac{10}{3} + \frac{8}{3} = \boxed{-\frac{2}{3}}$

e. $\int_1^4 3\sqrt{x}\,dx = \int_1^4 3x^{\frac{1}{2}}\,dx = \dfrac{3x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_1^4$

$= 2x^{\frac{3}{2}}\Big|_1^4 = 2(4)^{\frac{3}{2}} - 2(1)^{\frac{3}{2}}$

$= 2(8) - 2(1) = \boxed{14}$

Evaluating an Integral Involving Absolute Value

Example: Evaluate.

$2x-1=0 \Rightarrow x=\frac{1}{2}$

vertex: $(\frac{1}{2}, 0)$

$$\int_0^2 |2x-1|\,dx =$$

$|2x-1| \Rightarrow y=2x-1$ or $y=-(2x-1)$

$y=-2x+1$

$$\int_0^{\frac{1}{2}}(-2x+1)\,dx + \int_{\frac{1}{2}}^{2}(2x-1)\,dx$$

$$\left[\frac{-2x^2}{2}+x\right]_0^{\frac{1}{2}} + \left[\frac{2x^2}{2}-x\right]_{\frac{1}{2}}^{2}$$

$$\left[-x^2+x\right]_0^{\frac{1}{2}} + \left[x^2-x\right]_{\frac{1}{2}}^{2}$$

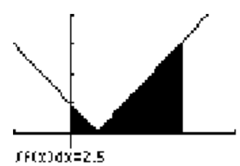
$$\left[\left(-\left(\tfrac{1}{2}\right)^2+\tfrac{1}{2}\right)-0\right] + \left[\left(2^2-2\right)-\left(\left(\tfrac{1}{2}\right)^2-\tfrac{1}{2}\right)\right]$$

$$-\frac{1}{4}+\frac{1}{2} \qquad + \left[2-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$\frac{1}{4}$$

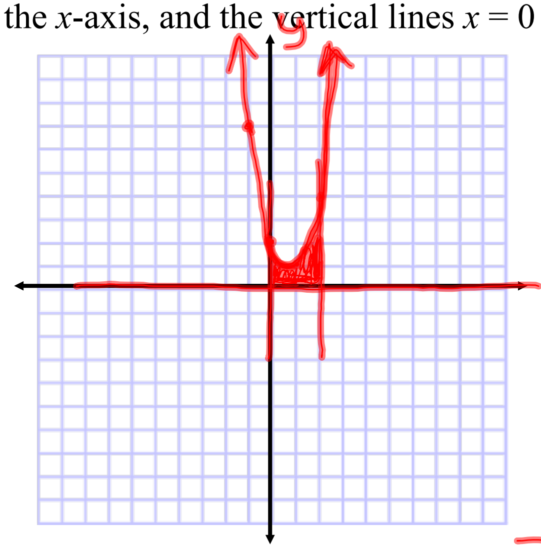
$$\frac{1}{4} + \left(2-\left(-\frac{1}{4}\right)\right)$$

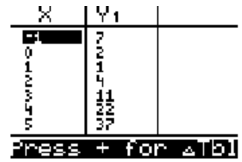
$$\frac{1}{4}+2+\frac{1}{4} = \boxed{2.5 \text{ or } \frac{5}{2}}$$



Example:

Find the area of the region bounded by the graph of $y = 2x^2 - 3x + 2$, the x-axis, and the vertical lines $x = 0$ and $x = 2$.





X	Y₁
-1	7
0	2
1	1
2	4
3	11
4	22
5	37

Press + for ΔTbl

$$\int_0^2 (2x^2 - 3x + 2)\,dx$$

$$= \left[\frac{2x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^2$$

$$= \left(\frac{2(2)^3}{3} - \frac{3(2)^2}{2} + 2(2)\right) - 0$$

$$= \boxed{\frac{10}{3}}$$