5.3 Integration and Riemann Sums

A tank is being filled with water using a pump that slows down as it runs. The table below gives the rate at which the pump pumps at ten-minute intervals. If the tank is initially empty, how many gallons of water are in the tank after 90 minutes?

<table>
<thead>
<tr>
<th>Elapsed time (min)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate (gal/min)</td>
<td>42</td>
<td>40</td>
<td>38</td>
<td>35</td>
<td>35</td>
<td>32</td>
<td>28</td>
<td>20</td>
<td>19</td>
<td>15</td>
</tr>
</tbody>
</table>

Left Hand Sum =

\[10(42) + 10(38) + 10(35) + 10(35) + 10(32) + 10(28) + 10(20) + 10(19)\]

= 2890 gallons
(an overestimate)

Right Hand Sum =

\[10(40 + 38 + 35 + 35 + 32 + 28 + 20 + 19 + 15)\]

= 2620 gallons
(an underestimate)
Left Riemann Sum
(Left Rectangular Approximation Method)

Find the LRAM when $n = 6$

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>20</td>
<td>13</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>45</td>
</tr>
</tbody>
</table>

$$LRAM = 2(20 + 13 + 10 + 20 + 30 + 40) = 266$$
Right Riemann Sum
(Right Rectangular Approximation Method)

Find the RRAM when \( n = 6 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>20</td>
<td>13</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>45</td>
</tr>
</tbody>
</table>

RRAM = \( 2(13 + 10 + 20 + 30 + 40 + 45) \)

= \( 316 \)
Midpoint Riemann Sum
(Midpoint Rectangular Approximation Method)

Find the MRAM. Before you start, think about what n should be.

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>20</td>
<td>13</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>45</td>
</tr>
</tbody>
</table>

\[\text{MRAM} = 4(13 + 20 + 40) = 292\]
Trapezoidal Rule

Use the trapezoidal rule to find the area under the curve from 2 to 14.

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>20</td>
<td>13</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>45</td>
</tr>
</tbody>
</table>

\[ A = \frac{1}{2} h(b_1 + b_2) \]
\[ A = \frac{1}{2} b(h_1 + h_2) \]

\[ \text{Trapezoidal Rule:} \]
\[ \frac{1}{2}(a)(20 + 13) + \frac{1}{2}(a)(13 + 10) + \frac{1}{2}(a)(10 + 20) \]
\[ + \frac{1}{2}(a)(20 + 30) + \frac{1}{2}(a)(30 + 40) + \frac{1}{2}(a)(40 + 45) \]
\[ = 291 \Rightarrow \text{Trap is the average of LRAM and RRAM} \]
Archimedes' Method of Exhaustion

By increasing $n$ (number of sides of the polygon inscribed in the circle), the approximation for area of the circle becomes closer and closer to the actual area.
LRAM

RRAM

MRAM

Trapezoid Method

↑

the best

(∞ many trapezoids)

is ideal.
Definite Integrals = Area under the curve

\[ \int_{a}^{b} f(x) \, dx = s \]

- **Integration symbol**
- **Integrand**
- **Change in x**
- **Upper limit of integration**
- **Lower limit of integration**
- **Area underneath the curve between \( a \) and \( b \)**
If a train traveled 75 miles per hour for 2 hours, how far did the train travel?

This problem is not as easy if the velocity is not a straight line (if it is a curve)

The area underneath the curve of the velocity function gives you distance traveled

\[
\int v(t)\,dt = s(t) \quad \int a(t)\,dt = v(t) \quad \text{\textit{\(s\text{\_velocity} = \text{position}\)}} \quad \text{\textit{\(s\text{\_acceleration} = \text{velocity}\)}}
\]
Definite Integrals on the Calculator:

The figure below shows the graph of \( f(x) = \sqrt{x} \)

Find LRAM, RRAM, MRAM, and the trapezoidal approximation of area to approximate the integral \( \int_{1}^{4} \sqrt{x} \, dx \)

Let \( n = 3 \). Then find the integral on your calculator.

\[
\text{LRAM} = \frac{1}{3} \left( \sqrt{1} + \sqrt{2} + \sqrt{3} \right) = 2.15
\]

\[
\text{RRAM} = \frac{1}{3} \left( \sqrt{2} + \sqrt{3} + \sqrt{4} \right) = 3.65
\]

\[
\text{MRAM} = \frac{1}{3} \left( \sqrt{1.5} + \sqrt{2.5} + \sqrt{3.5} \right) = 4.68
\]

\[
\text{Trap} = \frac{1}{3} \left( \sqrt{1} + 2 \left( \sqrt{2} + \sqrt{3} + \sqrt{4} \right) \right) = \frac{1}{3} \left( 1 + \sqrt{2} + \sqrt{3} + \sqrt{4} \right) = 4.68
\]

Definite Integrals on the Calculator:

\[ \text{MATH} \# 9 \]

\[ \int_{1}^{4} \sqrt{x} \, dx \]

Or

\[ \fnInt(\sqrt{x}, x, 1, 4) \]
This table shows the velocity of a model train moving along the track for ten seconds.

<table>
<thead>
<tr>
<th>Time (secs):</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>v(t) (cm/sec):</td>
<td>0</td>
<td>9</td>
<td>17</td>
<td>8</td>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>

Find the LRAM when n = 5

\[ 2 \left( 0 + 9 + 17 + 8 + 4 \right) = 76 \text{ cm} \]

Find the RRAM when n = 5

\[ 2 \left( 9 + 17 + 8 + 4 + 11 \right) = 98 \text{ cm} \]

What about the MRAM?

Can't do without knowing exact midpts.

The Trapezoidal Rule?

\[ \frac{1}{6} \left[ \left( 0 + 9 \right) + \left( 9 + 17 \right) + \left( 17 + 8 \right) + \left( 8 + 4 \right) + \left( 4 + 11 \right) \right] = 87 \text{ cm} \]
On HW Packet:

③ Add this sentence:

For $0 < t < 12$, the graph of $r$ is concave down.