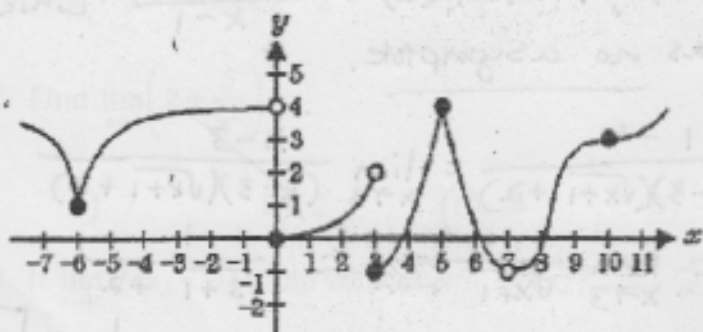# AP Calculus AB
Limits and Continuity Practice

Name _Key_

Date _____ Pd _____

#1 – 7 Multiple Choice:

The figure below shows the graph of f. Use this figure to answer questions #1 – 4.

1. $\lim\limits_{x \to 7} f$ is

     A) 1   B) 2   (C) –1   D) 4   E) 0

2. $\lim\limits_{x \to -6} f$ is

     (A) 1   B) 2   C) –1   D) 6   E) No limit

3. $\lim\limits_{x \to 0^-} f$ is

     A) 1   B) –1   C) 0   (D) 4   E) No limit

4. $\lim\limits_{x \to -6^-} f$ is

     (A) 1   B) 2   C) –1   D) 4   E) 6

5. $\lim\limits_{x \to 0^-} \dfrac{1}{x}$ is

     A) 1   B) 2   (C) $-\infty$ D) 0   E) ∞

6. The graph of which equation listed below has an asymptote of $x = 1$?

     A) $y = \dfrac{x-1}{x^2+1}$ B) $y = \sin x$ (C) $y = \dfrac{5x}{x^2-3x+2}$ D) $y = \cos x$ E) $y = \dfrac{x^2-4x+3}{x-1}$

$(x-2)(x-1)$

$\dfrac{(x-3)(x-1)}{x-1}$ hole @ $x = 1$

7. $\lim\limits_{x \to 2^+} \dfrac{x|x-2|}{x-2}$ is $\lim\limits_{x \to 2^+} x \cdot \lim\limits_{x \to 2^+} \dfrac{|x-2|}{x-2} = (2)(1) = \boxed{2}$

     A) –2   B) 4   C) 1   (D) 2   E) 0

8. Determine whether or not the following statement is true or false. If it is false, explain why and give a counterexample.

Statement: If $p(x)$ is a polynomial, then the function $f(x) = \dfrac{p(x)}{x-1}$ has a vertical asymptote at $x = 1$.

False; $f(x)$ could have a removable discontinuity.
Counterexample: If $p(x) = (x-1)\cdot(x+3)$, then $f(x) = \dfrac{(x-1)(x+3)}{x-1}$ which has no asymptote.

9. Find the limit. $\displaystyle\lim_{x \to 3} \dfrac{\sqrt{x+1}-2}{x-3}$

$$\lim_{x \to 3} \frac{(\sqrt{x+1}-2)}{(x-3)} \cdot \frac{(\sqrt{x+1}+2)}{(\sqrt{x+1}+2)} = \lim_{x \to 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x \to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x \to 3} \frac{1}{\sqrt{x+1}+2} = \frac{1}{\sqrt{3+1}+2}$$

$$= \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

10. For the function below:
   a) Find the x-values (if any) at which $f(x)$ is **not** continuous. Justify your answer(s).
       *Which of the discontinuities are removable? Which are not?
   b) Name any vertical asymptotes and verify using limits.
   c) Name any horizontal asymptotes and verify using limits.

Function: $f(x) = \dfrac{2x^2 - 7x - 15}{x^2 - x - 20} = \dfrac{(2x+3)(x-5)}{(x+4)(x-5)} = \dfrac{2x+3}{x+4}$

a) $f(x)$ is not continuous at $x = 5$ because $f(5)$ is undefined.

   $f(x)$ is not continuous at $x = -4$ because $f(-4)$ is undefined.

   $x = 5$ is removable discontinuity
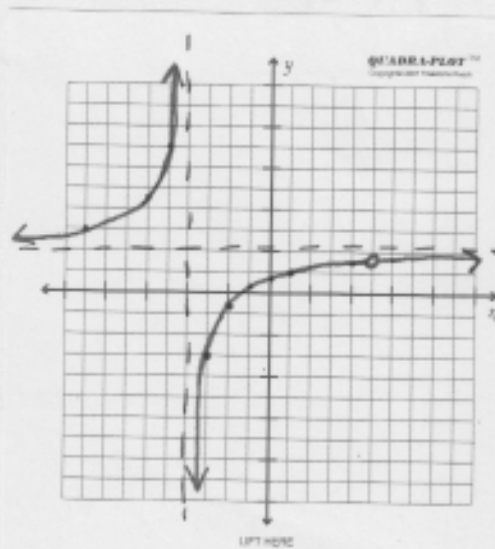 $x = -4$ is nonremovable discontinuity

b) $x = -4$ is a vertical asymptote

   $\displaystyle\lim_{x \to -4^-} f(x) = \infty$

   $\displaystyle\lim_{x \to -4^+} f(x) = -\infty$

c) $y = 2$ is a horizontal asymptote.

   $\displaystyle\lim_{x \to -\infty} f(x) = 2$

   $\displaystyle\lim_{x \to \infty} f(x) = 2$

11. What is $\lim\limits_{x\to 3}\dfrac{x^2-1}{x^2+4x-5}$? $=\dfrac{3^2-1}{3^2+4(3)-5}=\dfrac{9-1}{9+12-5}=\dfrac{8}{16}=\boxed{\dfrac{1}{2}}$

   A) $\dfrac{1}{5}$ (B) $\dfrac{1}{2}$ C) 1   D) 5   E) 0

12. Find $\lim\limits_{x\to 0}\left(2+\dfrac{5}{x^2}\right)$.

   A) 7   B) 2   (C) ∞ D) $-\infty$ E) None of these

13. If $\lim\limits_{x\to 10}f(x)=20$, then what is $\lim\limits_{x\to 10}2f(x)$? $=2\lim\limits_{x\to 10}f(x)=2(20)=\boxed{40}$

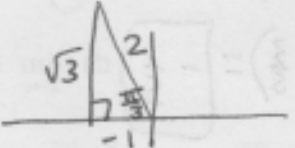   A) 5   B) 10   (C) 40   D) 80   E) None of these

14. Let $f(x)=\dfrac{1}{x+1}$ and $g(x)=x^2-5$. Find all values of x for which $f(g(x))$ is discontinuous.

   A) -1 B) $-1,\pm\sqrt{5}$ C) $\pm\sqrt{5}$ (D) $-2,2$ E) None of these

$f(g(x))=\dfrac{1}{x^2-5+1}=\dfrac{1}{x^2-4}=\dfrac{1}{(x+2)(x-2)}$
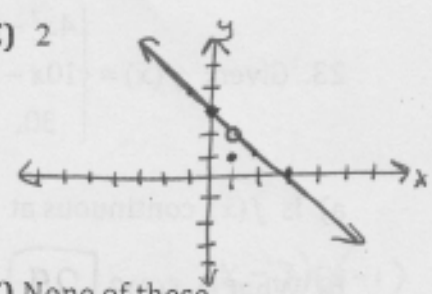
15. Find all vertical asymptotes of $g(x)=\dfrac{x+1}{x^2-1}$. $=\dfrac{\cancel{x+1}}{\cancel{(x+1)}(x-1)}$

   A) $x=-1,x=1$ B) $x=-1$ (C) $x=1$ D) $y=1$ E) None of these

16. Find $\lim\limits_{x\to 2}\sec\dfrac{\pi x}{3}$ $=\sec\dfrac{2\pi}{3}$ $\sec\dfrac{2\pi}{3}=\dfrac{hyp}{adj}=\dfrac{2}{-1}=\boxed{-2}$

   (A) -2 B) $\dfrac{2}{\sqrt{3}}$ C) $-\dfrac{\sqrt{3}}{2}$ D) $\dfrac{1}{2}$ E) 2

17. Find $\lim\limits_{x\to 1}f(x)$ if $f(x)=\begin{cases}3-x, & x\neq 1\\ 1, & x=1\end{cases}$ (Graphing may help)

   (A) 2   B) 1   C) $\dfrac{3}{2}$ D) Does not exist   E) None of these

18. At which values of x is $f(x)=\dfrac{x^2-2x-3}{x-2}$ discontinuous? $=\dfrac{(x+1)(x-3)}{x-2}$

   (A) 2   B) $-1,2,3$ C) 1   D) $-1,\dfrac{3}{2},2,3$ E) None of these

19. Find $\lim\limits_{x \to -1} \dfrac{3x^2 - 2x - 1}{x^2 + 1} = \dfrac{3(-1)^2 - 2(-1) - 1}{(-1)^2 + 1} = \dfrac{3(1) + 2 - 1}{1 + 1}$

$$= \frac{4}{2} = \boxed{2}$$

20. Find $\lim\limits_{x \to 2^-} \dfrac{|x - 2|}{x - 2} = \boxed{1}$

21. Find the constant(s) c for which the function $f(x)$ is continuous on $(-\infty, \infty)$.

$f(x) = \begin{cases} c^2 - x^2 & \text{if } x < 2 \\ 2(x + c) & \text{if } x \geq 2 \end{cases}$

$c^2 - x^2 = 2(x + c)$ when $x = 2$

$c^2 - (2)^2 = 2(2 + c)$

$c^2 - 4 = 4 + 2c$

$c^2 - 2c - 8 = 0$

$(c - 4)(c + 2) = 0$

$$\boxed{c = 4, \quad c = -2}$$

22. Given $\lim\limits_{x \to c} f(x) = \dfrac{1}{3}$ and $\lim\limits_{x \to c} g(x) = -\dfrac{3}{5}$, find:

a) $\lim\limits_{x \to c} [5g(x)] = 5\left(-\dfrac{3}{5}\right) = \boxed{-3}$

b) $\lim\limits_{x \to c} [f(x) + g(x)] = \dfrac{1}{3} - \dfrac{3}{5} = \dfrac{5}{15} - \dfrac{9}{15}$

$$= \boxed{\dfrac{-4}{15}}$$

c) $\lim\limits_{x \to c} [f(x)g(x)] = \left(\dfrac{1}{3}\right)\left(-\dfrac{3}{5}\right) = \boxed{-\dfrac{1}{5}}$

d) $\lim\limits_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{\frac{1}{3}}{-\frac{3}{5}} = \dfrac{1}{3} \cdot \dfrac{5}{-3}$

$$= \boxed{-\dfrac{5}{9}}$$

23. Given: $f(x) = \begin{cases} 4x^2 - 2x, & x < 3 \\ 10x - 1, & x = 3 \\ 30, & x > 3 \end{cases}$

a) Is $f(x)$ continuous at $x = 3$? $\boxed{\text{No}}$

b) What is $f(3)$? $\boxed{29}$

c) What is $\lim\limits_{x \to 3^-} f(x)$? $\boxed{30}$

d) What is $\lim\limits_{x \to 3^+} f(x)$? $\boxed{30}$

e) What is $\lim\limits_{x \to 3} f(x)$? $\boxed{30}$