The slope of a curve y = f(x) at the point x = a is

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

When it exists, this limit is called "the derivative of f

when it exists, this limit is called the derivation of a
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

f'(a) is read as "the derivative of f with respect to a"

This can also be written as:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

How do derivatives work and how are they used?

Example: Find the derivative of the function.

$$f(x) = x^{3} - 6x + 3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^{3} - 6(x+h) + 3 - (x^{3} - 6x + 3)}{h}$$

$$= \lim_{h \to 0} \frac{x^{3} + 3x^{2}h + 3xh^{2} + h^{3} - 6x - 6h + x + x^{3} + 6x + x^{4}}{h}$$

$$= \lim_{h \to 0} 3x^{2} + 3xh + h^{2} - 6 = 3x^{2} - 6$$

$$f'(x) = 3x^{2} - 6$$

The derivative of f is the <u>expression</u> we use to find m_{tan} at any point.

Find
$$f'(1)$$
. = $3(1)^2 - 6 = (-3) \rightarrow m_{tan} = 1$

This is the slope of the tangent line at x = 1.

There are many ways to write the derivative of y = f(x)

Symbol	Read as:
f'(x)	"f prime x" or "the derivative of f with respect to x"
y'	"½ prime"
$\frac{dy}{dx}$	"dee why dee ecks" or "the derivative of y with respect to x"
$\frac{df}{dx}$	"dee eff dee ecks" or "the derivative of f with respect to x"
$\frac{d}{dx}f(x)$	"dee dee ecks of f or "the derivative of of ecks" f of x " $(d dx \text{ of } f \text{ of } x)$

Be Careful!!!

 $dx \, \underline{\text{does not}} \, \underline{\text{mean}} \, d \, \text{times} \, x$

 $dy \underline{\operatorname{does}} \underline{\operatorname{not}} \operatorname{mean} d \operatorname{times} y$

$$\frac{d}{dx}f(x)$$
 does NOT mean $\frac{d}{dx}$ times $f(x)$

The Derivative Function

- The domain of f' is the set of points in the domain f for which the limit exists
- If f'(x) exists, we say that f has a derivative or "f is differentiable"
- A function that is differentiable at every point in its domain is a differentiable function
- Functions can be differentiable at certain points

One Sided Derivatives

A function y = f(x) is differentiable on a closed interval [a,b] if it has a derivative at every interior point of the interval, and if the limits exist at the endpoints.

$$\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$
 the right hand derivative of a

$$\lim_{h \to 0^{-}} \frac{f(b+h) - f(b)}{h}$$
 the left hand derivative of b

- Remember a function has a two sided derivative at a point iff (if and only if) the two are defined and equal at that point, but the function must be continuous at the point.
- One sided limits can differ at a point and have no derivative there!

Theorem 3.2 – The Constant Rule

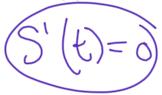
The derivative of a constant function is 0. That is, if c is a real number, then:

$$\frac{d}{dx}[c]=0$$

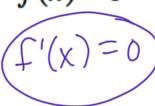
Example 1: Find the derivative of the following functions.

a.
$$y = 7$$

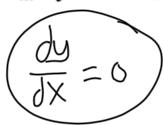
c.
$$s(t) = -3$$



b.
$$f(x)$$
=



d.
$$y = k\pi^2$$
, \underline{k} is constant



Theorem 3.3 - The Power Rule

If n is a real number, then the function $f(x) = x^n$ is differentiable and:

$$\frac{d}{dx} \left[x^n \right] = nx^{n-1}$$

For f to differentiable at x = 0, n must be a number such that x^{n-1} is defined on an interval containing 0.

Example 2: Find the derivative of the following functions.

a.
$$f(x) = x^3$$

$$f'(x) = 3x^2$$
c. $y = \frac{1}{x^2} = x^{-2}$

$$\frac{dy}{dx} = -2x^{-3}$$

$$= \begin{bmatrix} -2 \\ x^3 \end{bmatrix}$$

b.
$$g(x) = \sqrt[3]{x}$$
 $g(x) = x^{\frac{1}{3}}$
 $g(x) = x^{\frac{1}{3}}$
 $g'(x) = \frac{1}{3}x^{\frac{1}{3}-1}$
 $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$
d. $h(x) = x$

$$|x|^{0}$$

$$|x|^{0}$$

Example 3: Find the slope of $f(x) = x^4$ when:

a.
$$x = -1$$

$$f'(-1) = 4(-1)^3 = (-4)$$

b.
$$x = 0$$
 $f'(x) = 4x^3 \implies f'(0) = 4(0)^3 = 0$

c.
$$x = 1$$
 $f'(x) = 4x^3 \longrightarrow f'(1) = 4(1)^3 = 4$

Example 4: Find the equation of the tangent line to the graph of $f(x) = x^2$ when x = -2. m = -9

$$M_{tan} = f'(x) = 2x$$

$$m_{tan} = f'(-2) = 2(-2) = -4$$
 $(y - 4 = -4(x+2))$
 $e_{x=-2}$

$$m = -4$$
 $(-2, 4)$
 $(-4 = -4(x+2))$