11 5 Recursion and Iteration

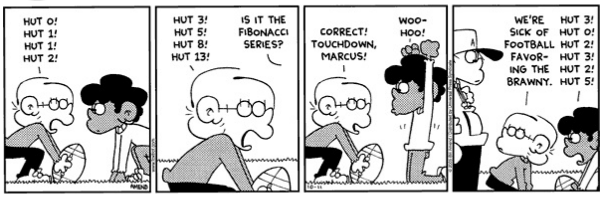
LESSON 11-5 Recursion and Iteration

1 Special Sequences Notice that every term in the list of ancestors is the sum of the previous two terms. This special sequence is called the **Fibonacci sequence**, and it is found in many places in nature. The Fibonacci sequence is an example of a **recursive sequence**. In a recursive sequence, each term is determined by one or more of the previous terms.

The formulas you have used for sequences thus far have been explicit formulas. An **explicit formula** gives a_n as a function of n, such as $a_n = 3n + 1$. The formula that describes the Fibonacci sequence, $a_n = a_{n-2} + a_{n-1}$, is a **recursive formula**, which means that every term will be determined by one or more of the previous terms. An initial term must be given in a recursive formula.

Example: Fibonacci 0, 1, 1, 2, 3, 5, 8, 13, 21, ...

http://www.youtube.com/watch?v=P0tLbl5LrJ8



Recursive formulas for Sequences:

Arithmetic Sequence: $a_n = a_{n-1} + d$

Geometric Sequence: $a_n = r \bullet a_{n-1}$

Example 1: Find the first five terms of the sequence in which $\underline{a_1 = -3}$ and $\boxed{a_{n+1} = 4a_n - 2}$ if $n \geq 1$

$a_2 = a_{1+1} = 4a_1 - 2 = 4(-3) - 2 = -14$

$a_3 = a_{2+1} = 4(a_2) - 2 = 4(-14) - 2 = -58$

$a_4 = a_{3+1} = 4(a_3) - 2 = 4(-58) - 2 = -234$

$a_5 = a_{4+1} = 4(a_4) - 2 = 4(-234) - 2 = -938$

$\boxed{-3, -14, -58, -234, -938}$

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You Try: Find the first 5 terms of the sequences:

a) $a_1 = 4, a_{n+1} = 2a_n + 1$

$a_2 = 9$

$a_3 = 19$

$a_4 = 39$

$a_5 = 79$

b) $a_1 = 16; a_n = a_{n-1} + (n+3)$

$a_1 = 16$

$a_2 = a_{2-1} + (2+3) = 16 + 5 = 21$

$a_3 = a_2 + (3+3) = 21 + 6 = 27$

$a_4 = a_3 + (4+3) = 27 + 7 = 34$

$a_5 = a_4 + (5+3) = 34 + 8 = 42$

16, 21, 27, 34, 42

Example 2: Write a recursive formula for each sequence.

a) 2, 10, 18, 26, 34, ...

+8

$a_n = a_{n-1} + 8, \; a_1 = 2$

$a_{n+1} = a_n + 8, \; a_1 = 2$

b) 16, 56, 196, 686, 2401, ...

$r = \frac{56}{16} = \frac{7}{2}$ or 3.5

$a_n = 3.5a_{n-1}, \; a_1 = 16$

$a_{n+1} = 3.5a_n, \; a_1 = 16$

c) $a_4 = 108$ and r = 3

$a_n = 3a_{n-1}, \; a_1 = 4$

$\frac{r^4}{r^1} = \frac{108}{a_1}$

$r^3 = \frac{108}{a_1}$

$3^3 = \frac{108}{a_1}$

$\frac{27a_1 = 108}{27}$

You Try: Write a recursive formula for each sequence.

a) 8, 17, 26, 35, 44, ...

$+9$

$\boxed{a_n = a_{n-1} + 9, \; a_1 = 8}$

b) $a_3 = 16$ and $r = 4$

$a_2 = 4$

$a_1 = 1$

$\boxed{a_n = 4a_{n-1}, \; a_1 = 1}$

Example 3: Find the first three iterates of $f(x) = 5x + 4$
for an initial value of x_o=2.

$x_1 = f(x_0) = 5(2) + 4 = 14$

$x_2 = f(x_1) = 5(14) + 4 = 74$

$x_3 = f(x_2) = 5(74) + 4 = 374$

You Try: Find the first three iterations of each function, using the given initial value.

$$f(x) = -x^2 - 2x + 1, \cancel{a_1}\,x_0 = -2$$

$f(x_0) = x_1 = -(-2)^2 - 2(-2) + 1$

$x_1 = -4 + 4 + 1 = 1$

$f(x_1) = x_2 = -(1)^2 - 2(1) + 1 = -2$

$f(x_2) = x_3 = -(-2)^2 - 2(-2) + 1 = 1$

$1, -2, 1$

Example 4:

FINANCIAL LITERACY Nate had \$15,000 in credit card debt when he graduated from college. The balance increased by 2% each month due to interest, and Nate could only make payments of \$400 per month. Write a recursive formula for the balance of his account each month. Then determine the balance after five months.

$a_n = (a_{n-1})1.02 \quad -400$

$\boxed{14{,}479.60}$

a_2 1 month $\overset{a_1}{15{,}000}(1.02)-400$

14,900

a_3 2 month $14{,}900(1.02)-400$

Example 5: Write a recursive formula for the following

a) 2, $\times 2$ 4, $\times 4$ 16, $\times 16$ 256 . . .

$a_n = (a_{n-1})^2, \ a_1 = 2$

b. 1, 2, 2, 4, 8, 32 . . .

$a_n = (a_{n-1})(a_{n-2}), \ a_1 = 1 + a_2 = 2$

c) 4, 8, 10, 14, 19, 26 . . .